Astrophys Space Sci (2016) 361:130
DOI 10.1007/s10509-016-2712-5

ORIGINAL ARTICLE

Localized structures in complex plasmas in the presence of a magnetic field

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Received: 6 June 2015 / Accepted: 8 March 2016

Abstract In this work, the general framework in which fits our investigation is that of modeling the dynamics of dust grains therein dusty plasma (complex plasma) in the presence of electromagnetic fields. The generalized discrete complex Ginzburg-Landau equation (DCGLE) is thus obtained to model discrete dynamical structure in dusty plasma with Epstein friction. In the collisionless limit, the equation reduces to the modified discrete nonlinear Schrödinger equation (MDNLSE). The modulational instability phenomenon is studied and we present the criterion of instability in both cases and it is shown that high values of damping extend the instability region. Equations thus obtained highlight the presence of soliton-like excitation in dusty plasma. We studied the generation of soliton in a dusty plasma taking in account the effects of interaction between dust grains and theirs neighbours. Numerical simulations are carried out to show the validity of analytical approach.

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Keywords Dusty plasma · Modulational instability · Multiple time scale · Modified discrete nonlinear Schrödinger equation · Envelope solitons

1 Introduction

In the recent decades, the study of wave propagation in dusty plasmas has seen spectacular growing interest among plasma physicists, not just because of the omnipresent of dust in our universe, but also because of its special role in explaining some collective processes (coherent structures) in laboratory plasmas, astrophysical and space environment (space plasmas) in Shukla and Mamun (2002); Saini et al. (2014); Siminos et al. (2014). A dusty (complex) plasma is a multicomponent system consisting of electrons, ions, charged mesoscopic particles (dust grains) and neutral atoms or molecules (Rahman et al. 2015; Shukla and Eliasson 2009; Morfill and Ivlev 2009; Meyer-Vernet et al. 2015). Interest in this 'unusual' state of matter stems from the ubiquity with which it is found in the laboratory, in space, and in astrophysics, such as cometary tails, planetary rings, solar and planetary nebulae, the lower ionosphere (mesosphere), atmospheric lighting and industrial plasma processing and nanomaterials fabrication devices (Merlino 2006; Amorim et al. 2015). Most of the theoretical works on wave propagation in plasma are focused on ion acoustic wave (IAW) mode (Alinejad et al. 2014; Jain et al. 2015; Hossen et al. 2015) and electron-acoustic wave (Rafat et al. 2015), but the more on the new acoustic-like oscillatory modes as well as dust acoustic (DA) waves and dust ion acoustic (DIA) waves (Duan et al. 2003; Rao and Yu 1990; Shukla and Silin 1992; Sultana et al. 2014).

The DA wave is a low-frequency, longitudinal wave characterized by propagating dust density compressions and rar-

Published online: 22 March 2016

efactions (Rao and Yu 1990). It is a sound wave propagating through the charged dust fluid, involving oscillations of the heavy dust grains. The interactions between the charged dust grains are mediated by the collective electric fields in the plasma. The existence of the DA wave was suggested almost some years ago by Padma Shukla at the First Capri Workshop on Dusty Plasmas in 1989. The detailed analysis of this dust wave was worked out by Rao, Shukla and Yu a few months later (Rao and Yu 1990). Shukla's contribution was in treating the dust as a separate fluid component which could support electrostatic waves of such a low frequency that the inertia of both the electrons and ions could be ignored (Boltzmann response). An intriguing aspect of the DA wave is that, due to light scattering from the grains, the waves could be seen propagating through the dust suspension with the naked eye (Barkan et al. 1995). The modulated wave packets and envelope solitary structure of DA wave in dusty or complex plasmas have been studied by Kourakis and Shukla (2004a, 2004b) and recently the effect of excess superthermal electrons on the modulational instability and envelope soliton modes were studied by Sultana and Kourakis (2011). The amplitude modulation of longitudinal (Kourakis and Shukla 2004e; Amin et al. 1998a, 1998b), transverse (vertical, off-plane) (Kourakis and Shukla 2004c) dust lattice waves and weakly nonlinearity (Bains et al. 2013) was recently considered. The aims of this work is to gain more insight in the generation of localized structures in a complex plasma considering the case with Epstein friction and the case of collisionless dust charged plasmas in the presence of variable electromagnetic field. The modulational instability (MI) criteria of dust lattice wave (DLW) and the transverse magnetized dust lattice oscillations is then investigated.

A link is made on the methodology, in a quite exhaustive manner, in close relation with previous works of Kourakis and Shukla (2004b), and always by regarding the particular features of DP crystals.

It is well known that as nonlinear-type system, our model (the modified discrete complex Ginzburg-Landau (MDCGL) equation) can present an instability that leads to the self-induced modulation of a plane wave of entry with the continuously generation of the patterns. The MI in DNLS-like lattices was first predicted in the first Brillouin zone (Christodoulides and Joseph 1988) and experimental observation of discrete MI has been reported for the first time by Meier et al. (2004). The study of MI is essential as it stands as a precursor phenomenon to the formation of discrete soliton (Braun and Kivshar 1998; Henning and Tsironis 1999). The MI appears in the same parameter region where solitons are observed (Rajib et al. 2015). Nowadays, the study of MI in discrete models such as the DNLS equation attracts the attention of the scientific community. Longitudinal excitations of localized small amplitude were also considered in Shukla and Mamun (2003), Kourakis and Shukla (2004d), and Melandso (1996) with a quasicontinuum description of the dust lattice. Small amplitude is found to be described by a Korteweg-de Vries equation for the density and Boussinesq equation for the longitudinal displacement of grains. In addition, experiments (Liu et al. 2003) highlights the fact that the high discreteness of dust crystals should play an important role in mechanisms such MI (response to an external excitation).

The present study is devoted to look for the best conditions under which different patterns wave emerges in our dusty plasma during the propagation of the plane wave, given a certain wave number in the system. The plan of this paper is as follows. In Sect. 2, we present the model of dusty plasmas. The semi-discrete approach is used and lead to the MDCGLE in the damping case and in the collisionless case, to MDNLSE (Ablowitz 1975). Next, the conditions under which the wave become stable/unstable for a small perturbation of amplitude are derived in both cases. In Sect. 3, we use the fourth-oder Runge–Kutta method to study numerically the generation of soliton-like object through our system.

2 Model and linear stability analysis

we consider a layer of identical charged dust grains of lattice constant r_0. The Hamiltonian of such a chain is of the form

$$H = \sum_n \frac{1}{2} M \left(\frac{dr_n}{dt} \right)^2 + \sum_{m \neq n} U(r_{nm}) + \Phi_{\text{ext}}(r_n), \tag{1}$$

and the motion of the nth dust grain in the transverse vertical, off-plane, z-direction, we have the equation of motion including dissipation of the nth grain caused by dust-neutral collisions (Kourakis and Shukla 2004g)

$$M \left(\frac{d^2 z_n}{dt^2} + \nu \frac{dz_n}{dt} \right) = -\sum_n \frac{\partial U_{nm}(r_{nm})}{\partial z_n} + F_{\text{ext}}(z_n), \tag{2}$$

where $F_{\text{ext}}(z_n)$ accounts for all external forces in the z-direction.

2.1 Equation of motion

Considering small displacement from the equilibrium position, we can develop the interaction potential $U(r)$ around the intergrain equilibrium position $(z_n = 0) l r_0 = |n - m| r_0$ between the lth neighbours $(l = 1, 2, 3, \ldots)$. Thus, by retaining only interaction between near neighbours $(l = 1)$ the equation of motion of the nth grain is as follow: From (2) we have:

$$M \left(\frac{d^2 z_n}{dt^2} + \nu \frac{dz_n}{dt} \right) = -\sum_n \frac{\partial U_{nm}(r_{nm})}{\partial z_n} + F_{\text{ext}}(z_n),$$

by the way, $-F_{\text{ext}}(r_n) = \partial\Phi_{\text{ext}}(r_n)/\partial r_n$, and,

$$\Phi_{\text{ext}}(r_n) = \Phi_0 + \Phi_{(1)}z_n + \frac{1}{2}\Phi_{(2)}z_n^2 + \frac{1}{3!}\Phi_{(3)}z_n^3 + \frac{1}{4!}\Phi_{(4)}z_n^4$$

Eq. (2) then become:

$$\ddot{z} + \nu\dot{z} + \omega_g^2 z_n + K_1 z_n^2 + K_2 z_n^3 = \omega_{0,T}^2(2z_n - z_{n-1} - z_{n+1}) + K_3\big[(z_{n+1} - z_n)^3 - (z_n - z_{n-1})^3\big], \tag{3}$$

where the characteristic frequency of transversal oscillations $\omega_{0,T}$ and the coefficient K_3 are (Kourakis and Shukla 2004f):

$$\omega_{0,T}^2 = -\left(\frac{U'(r_0)}{Mr_0}\right),$$

$$K_3 = -\frac{1}{2Mr_0^3}\big[U'(r_0) - r_0 U''(r_0)\big],$$

$$\omega_g^2 = \frac{\Phi_{(2)}}{M},$$

$$K_1 = \frac{\Phi_{(3)}}{2M}; \qquad K_2 = \frac{\Phi_{(4)}}{6M}$$

From the nonlinear equation (3), we search for small-amplitude time-periodic solutions. So, we will study the behaviour of this wave with the help of the discrete spatial Fourier transform:

$$z_n = \sum_{p=-\infty}^{+\infty} a_n^{(p)} e^{ip\omega_b t}; \tag{4}$$

where ω_b is the eigenfrequency of the fundamental model ($p = 1$) and, $a_n^{(p)}$ are time independent amplitude of the pth mode: $a_n(\epsilon^2 t)$; $a_n^{(p)} \sim \epsilon^p \Rightarrow \dot{a}_n \sim \epsilon^2 t$; $\ddot{a}_n \sim \epsilon^4 t$

After substituting (4) into (3) and cancelling first order harmonic, we have:

$$a_n = \sqrt{\frac{2\omega_b}{3K_2 + 6K_3 + \beta}} \times \psi_n \times \exp\left[i\frac{\omega_g^2 - \omega_b^2 - 2\omega_0^2}{2\omega_b}t\right], \tag{5}$$

where $\beta = -\frac{4K_1^2}{\omega_g^2} - \frac{2K_1^2}{\omega_g^2 - 4\omega_b^2 + 2i\nu\omega_b}$ and the dynamic of dust grain in our model is then described by the modified discrete complex Ginzburg-Landau like equation:

$$i\dot{\psi}_n + P(\psi_{n+1} + \psi_{n-1}) + S|\psi_n|^2\psi_n - R\big\{-2\psi_n\big(|\psi_{n+1}|^2 + |\psi_{n-1}|^2\big) - \psi_n^*\big(\psi_{n+1}^2 + \psi_{n-1}^2\big) + 2|\psi_n|^2(\psi_{n+1} + \psi_{n-1}) + \psi_n^2\big(\psi_{n+1}^* + \psi_{n-1}^*\big) + |\psi_{n+1}|^2\psi_{n+1} + |\psi_{n-1}|^2\psi_{n-1}\big\} - i\eta\psi_n = 0, \tag{6}$$

with the complexes parameters

$$P = \frac{\omega_0^2}{2\omega_b - i\nu} = \frac{2\omega_b\omega_0^2}{4\omega_b^2 + \nu^2} + i\frac{\nu\omega_0^2}{4\omega_b^2 + \nu^2} = P_r + iP_i, \tag{7}$$

$$S = \frac{2\omega_b}{2\omega_b - i\nu} = \frac{4\omega_b^2}{4\omega_b^2 + \nu^2} + i\frac{2\nu\omega_b}{4\omega_b^2 + \nu^2} = S_r + iS_i, \tag{8}$$

$$R = \frac{6K_3\omega_b}{(2\omega_b - i\nu)(3K_2 + 6K_3 + \beta)} = R_r + iR_i, \tag{9}$$

$$\eta = \frac{\nu(\theta + \omega_b)}{2\omega_b - i\nu} = \frac{\nu(\omega_g^2 + \omega_b^2 - 2\omega_0^2)}{4\omega_b^2 + \nu^2} + i\frac{\nu^2(\omega_g^2 + \omega_b^2 - 2\omega_0^2)}{2\omega_b(4\omega_b^2 + \nu^2)} = \eta_r + i\eta_i. \tag{10}$$

Others works with dust charge variation and Epstein friction in dusty plasma shown that the dynamics of wave was governed by a nonlinear Schrödinger equation modified by a dissipative term (Ghosh et al. 2011; Li 2014).

In our purpose, one can see that the effects of damping on the dust lattice waves and the transverse magnetized dust lattice oscillations reduce their dynamics to the MD-CGL equation, which is characteristic to dissipative systems. This equation is known in many systems (Boccaletti et al. 2000, 2002; Willame et al. 1991; Otsuka 1999) and is shown in this study to describe discrete dynamical structures in complex plasma. Its parameters are depicted versus the dust neutral collision (damping) term and Fig. 1 presents the shape of each of them. In general, the coefficient P_r accounts for the energy tunneling between adjacent elements of the dust lattice, while the imaginary term $P_i(> 0)$ stands for gain due to the coupling between neighboring sites of the dust lattice. The cubic nonlinearity coefficients are complex quantities which real parts (S_r and R_r) correspond to elastic collisions in the system. They correspond to the repulsive interaction for positives values ($S_r, R_r > 0$) and attractive interaction for negatives ones ($R_r < 0$). Next, the imaginary part (S_i and R_i) represent the cubic nonlinear loss (gain) if they take negative (positive) values (Borhanian 2013), which appear due to inelastic collisions. Finally, since $\eta_r, \eta_i > 0$ (Fig. 1) the coefficient η indicates the linear gain parameter due to the feeding strength from the thermal dust.

However, this equation reduces to the modified discrete nonlinear Schrödinger (MDNLS) equation in collisionless limit ($\nu = 0 \to \eta = 0$ and P, R and S are real parameters). We use theses equations to study the possibility of generation of localized structures in our complex plasma through modulational instability.

2.2 The modulational instability criterion

Since the modulates signals are used in engineering, they are very important due to their property of being transmit-

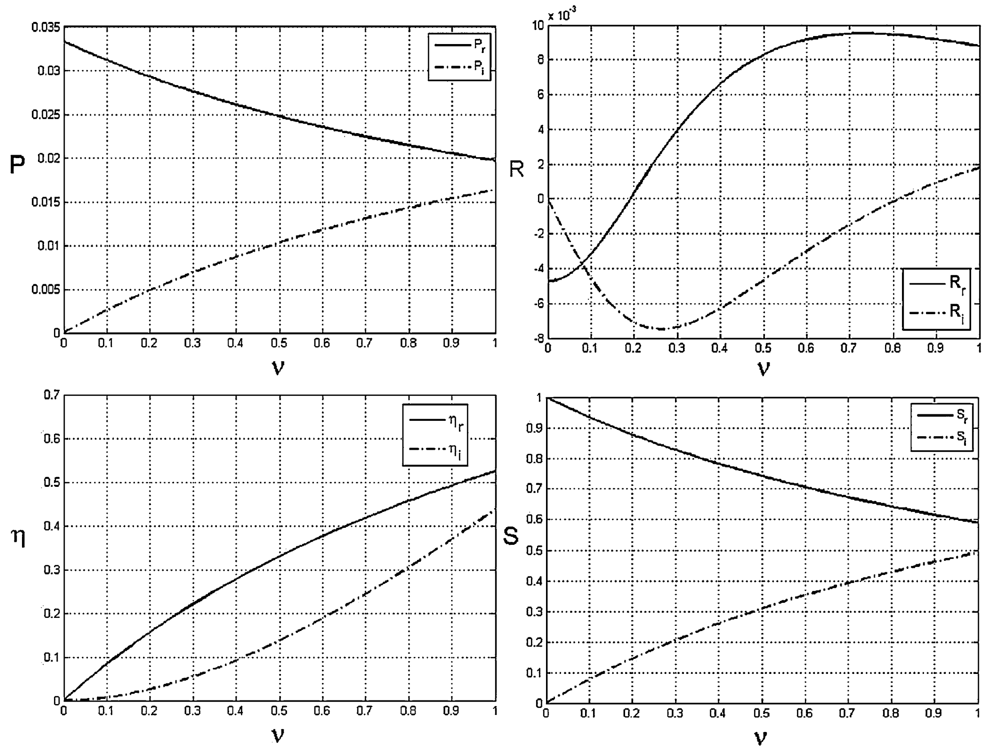

Fig. 1 Variation versus dust-neutral collision ν of the real (*continued lines*) and imaginary (*dotted lines*) part of different parameters of (6)

ting along much longer distance than non-modulated ones. Different approaches have been proposed to study the generation of modulated waves, and one of the ways to produce those with soliton-like objects forms is through modulational instability (MI). It is a result of the interplay between nonlinearity and dispersion and arises in continuous as well as in discrete systems. The MI is a general feature of continuum as well of discrete nonlinear wave equations and its demonstration spans a diverse set of disciplines, ranging from plasma physics, electrical transmission lines, nonlinear optics and DNA molecule, to cite just a few. In this section, we find the condition under which a uniform wave moving along the dusty plasma system will become modulationally stable or unstable to a small perturbation. We look for plane wave solutions of (6) in the form: $\psi_n = \psi_0 \exp[i(qn - \omega t)]$ we thus obtain the dispersion relation:

$$\omega + 2P_r \cos(q) + |\psi_0|^2 \{ S_r - R_r \big(8\cos(q) - 2\cos(2q) - 4 \big) \} + \eta_i = 0, \tag{11}$$

and

$$|\psi_0|^2 = \frac{-2P_i \cos(q) + \eta_r}{S_i + R_i(-8\cos(q) + 2\cos(2q) + 4)}. \tag{12}$$

Then, the modulational instability of a plane wave in such a dusty plasma lattice is investigated by studying the stability of its amplitude in the presence of sufficiently small perturbation so that one can linearize the equation of the envelope and the carrier wave. By the way, this phenomenon has been studied in various contexts such as fluid dynamics (the Benjamin-Feir instability) in Benjamin and Feir (1967); nonlinear optics and also in plasma physics (Tanuiti and Washimi 1992). Dauxois and Peyrard have shown that the modulational instability of a linear wave is a first step towards energy localization in nonlinear lattices (Dauxois and Peyrard 1993). The first step to probe some particular features of these excitations in our system is to introduce a small perturbation in the amplitude and in the phase, and look for the solution of (6). The modulational stability/instability of an extended nonlinear wave in such a system is related to the time evolution of a perturbed nonlinear wave of the form:

$$\psi_n = (\psi_0 + a_n) \exp\big[i(qn - \omega t + \phi_n) \big], \tag{13}$$

where ψ_0 is a constant amplitude of a plane wave. Replacing (13) in (6) and assuming $|\phi_n(t)| \ll (qn - \omega t)$ and also that $|a_n(t)| \ll \phi_0$, we obtain the following equation describing the evolution of the perturbations $a_n(t)$ and $\phi_n(t)$:

$$i\frac{\partial a_n}{\partial t} - \psi_0 \frac{\partial \phi_n}{\partial t} + P\{ \big[\big(a_{n+1} + a_{n-1} - 2a_n + i\psi_0(\phi_{n+1} + \phi_{n-1} - 2\phi_n) \big) \big] \cos(q) + \big[i(a_{n+1} - a_{n-1}) - \psi_0(\phi_{n+1} - \phi_{n-1}) \big] \sin(q) \} + 2\psi_0^2 R\{ \big[\big(a_{n+1} + a_{n-1} + a_n^* - a_n$$

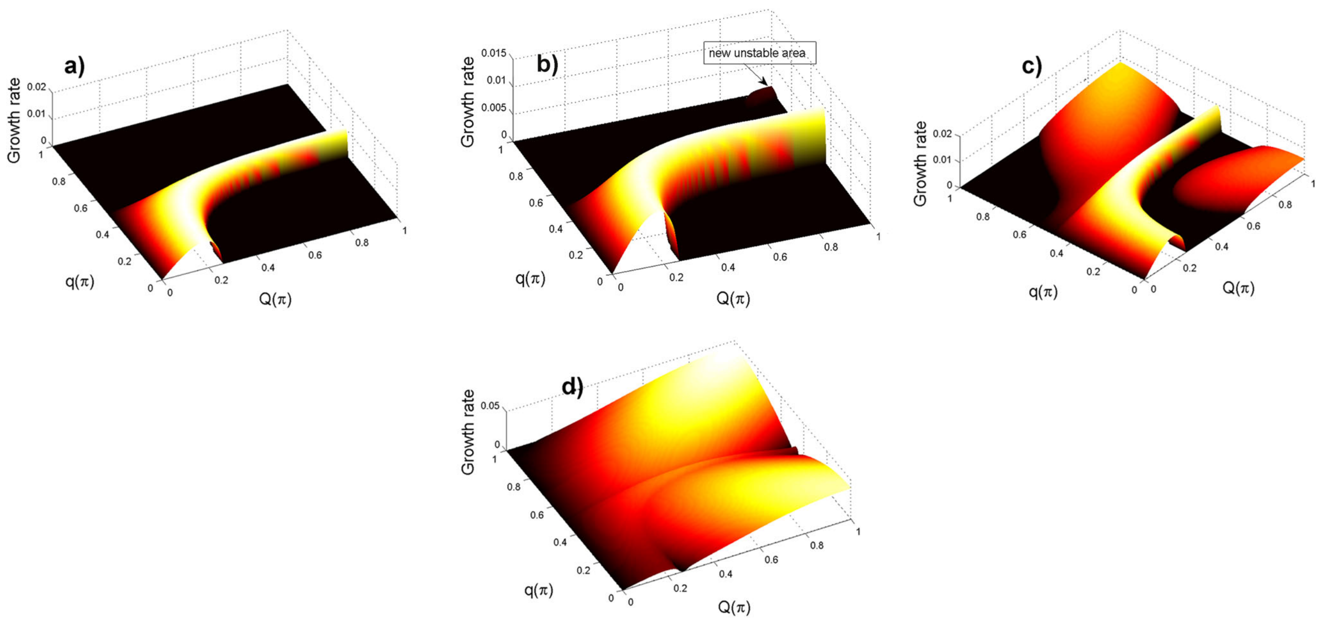

Fig. 2 Unstability (Color zone) and stability (Dark zone) diagrams in the (Q, q) plane with the parameters: $\omega_g = 1$; $K_1 = -0.5\omega_g^2$; $K_2 = 0.07\omega_g^2$; $K_3 = -0.005$; $\omega_b = 0.6$; $\omega_0 = 0.2$; (**a**) $\nu = 0$; (**b**) $\nu = 0.04$; (**c**) $\nu = 0.075$; (**d**) $\nu = 0.5$

$$
\begin{aligned}
&+ i\psi_0(\phi_{n+1} + \phi_{n-1} - 2\phi_n))\big]\cos(2q) \\
&+ \big[i(a_{n+1} - a_{n-1}) - \psi_0(\phi_{n+1} - \phi_{n-1})\big]\sin(2q) \\
&+ \big(a_{n+1} + a_{n-1} + a^*_{n+1} + a^*_{n-1}\big) \\
&- \big[2a_{n+1} + 2a_{n-1} + a^*_{n+1} + a^*_{n-1} + 2a^*_n \\
&+ i\psi_0(\phi_{n+1} + \phi_{n-1} - 2\phi_n)\big]\cos(q) \\
&+ 2\big[\psi_0(\phi_{n+1} - \phi_{n-1}) - i(a_{n+1} - a_{n-1})\big]\sin(q)\big\} \\
&+ S\psi_0^2\big(a_n + a^*_n\big) = 0. \qquad (14)
\end{aligned}
$$

Then we get the following equation describing the dynamics of the perturbation:

$$
\begin{aligned}
\phi_n &= \phi_1 e^{i(Qn+\Omega t)} + \phi_2 e^{-i(Qn+\Omega^* t)}, \\
a_n &= a_1 e^{i(Qn+\Omega t)} + a_2 e^{-i(Qn+\Omega^* t)},
\end{aligned}
\qquad (15)
$$

where (*) refers to complex conjugates of variables; Q and Ω, the wave number and the frequency of the perturbation, characterize linear properties; ϕ_1, ϕ_1, a_1 and a_2 are real constants.

Since it is known that the asymptotic behavior of the perturbation is related to the sign of the imaginary part of Ω (Nguenang et al. 2005; Daumont et al. 1997), one can look for the measure of the growth rate of the perturbation shown by (14) like $G(q, Q) = |\mathrm{Im}(\Omega)|$.

Substituting (15) into (14) yields to the following homogenous system of ϕ_1, ϕ_2, a_1 and a_2 such that: $Mu = 0$ with $u^{\perp} = [\phi_1, a_1, \phi_2, a_2]$ and the matrix M is defined by:

$$
M = \begin{pmatrix}
m_1 & m_2 - \Omega & 0 & m_4 \\
m_5 - \Omega & m_6 & 0 & m_8 \\
0 & m_{10} & m_{11} & m_{12} - \Omega \\
0 & m_{14} & m_{15} - \Omega & m_{16}
\end{pmatrix}, \qquad (16)
$$

and we have non trivial solutions by balancing the determinant of the matrix M with zero; this leads to the relation of dispersion:

$$
\begin{aligned}
&\Omega^4 + K_3(Q, q)\Omega^3 + K_2(Q, q)\Omega^2 + K_1(Q, q)\Omega \\
&\quad + K_0(Q, q) = 0, \qquad (17)
\end{aligned}
$$

where coefficients $K_i(Q, q)$, $(i = 0, \ldots, 3)$ and m_{ij}, $(i, j = 1, \ldots, 4)$ are real and expressed in the Appendix. This dispersion relation, which shows Ω as a function of the waves numbers q and Q, and system parameters, including the MI gain $G(q, Q) = |\mathrm{Im}(\Omega)|$, is then obtained from the condition of the existence of nontrivial solutions of the algebraic linear homogeneous system $\det(M) = 0$, which amounts to a quartic equation (17). After solving this condition of existence of nontrivial solutions numerically, we only keep the values of the wave numbers which give negative values of the imaginary part of Ω. Doing so, it allows to obtain by numerical computation the region (Fig. 2) where wave modulation is expected (colored zone) and stability region (Dark zone) diagrams in the (Q, q) plane. In these stability regions, at least one of the four growth rates has positive imaginary part.

Since our system is dissipative (non-conservative system), the counterbalance between loss and gain is added to

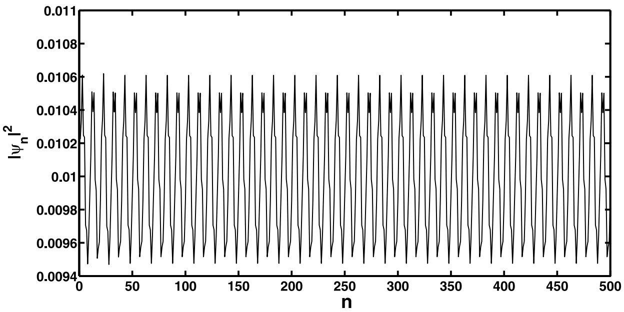

Fig. 3 Modulational stability. Same parameters as in Fig. 1, for the carrier wave and modulated wave number $q = 0.2\pi$; $Q = 0.9\pi$

the balance between nonlinearity and dispersion for formation of solitary waves which are called dissipative solitons in this case. Moreover, dissipative media has the capability to generate shock waves due to the interplay between dispersion and dissipation (Borhanian 2012). Thus, in Fig. 2, the corresponding gain shown in tridimensional, one observes that the growth rate of instability increases with the damping due to dust-neutral collision and high values of damping extend the instability region for high values of wave number Q of perturbation. This widening makes the choice of wave number for possible modulation very easy. This is a first confirmation of the possibility of MI in the system under our study. This increase of the growth rate shows the way through MI, which stands as a precursor phenomenon to formation of soliton, dissipative solitons could be localized in our dusty plasma due to Epstein damping ν.

3 Numerical simulations and discussions

The stability of a nonlinear plane wave with wave number q modulated by a small-amplitude wave of wave number Q is determined by the dispersion relation (17). Linear stability analysis doesn't predict well the long time evolution of a modulated nonlinear plane wave. Thus, to look for the validity of our analytical approach and the kind of dynamical patterns one might obtain in our dusty plasmas system under small perturbations, we carried out numerical simulations of the MDCGLE equation by using the fourth-order Runge–Kutta method. The initial conditions, typically at time $t = 0$, are coherently modulated plane waves of the form:

$$\psi_0\big(1 + \varepsilon\cos(Qn)\big)\exp\big(iqn + 0.01\cos(Qn)\big), \tag{18}$$

with $\varepsilon \ll 1$ and we have performed simulations with a lattice of 500 dust grain with periodic boundary conditions $\psi_n(t) = \psi_{n+N}(t)$ where N is the lattice length (in units of the lattice constant) (Natanzon et al. 2007). Figure 2 shows the instability diagram according to the stability limits of (17) and for wavenumbers labelled in black regions, we should obtain the MI phenomenon as theoretically predicted. So, we chose from Fig. 2 a set of wavenumbers q and Q for numerical studies. We first consider the case of points labelled in the black region, corresponding to modulational stability. As specific example we use the point $q = 2\pi/10$ and $Q = 9\pi/10$ and the initial condition is introduced in the system. One obtains that no instability is observed, the amplitude remains stable during the propagation as shown is Fig. 3.

The wave pattern displayed by the set of the preceding wave number is that of a plane wave with a sinusoidal form, with a constant amplitude that is not sensitive to any modulation as the time increases. Therefore, the system is said to be stable under the corresponding modulation.

Now, we take in Fig. 2 a set of wave numbers. One obtains that an initial plane wave with a sinusoidal form, with a constant amplitude become sensitive to modulation as the time increases: the wave displays an oscillating and breathing wave behavior. Figure 4 presents how the amplitude of the wave generated by wave motion is modulated in the form of a train of small amplitude with short wavelength.

Figure 4: the panels show how the initial plane solution wave breaks into wave train which has the shape of a soliton-like object in the dust lattice, as predicted by the analytical predictions, for parameters of diagram (Fig. 2).

4 Conclusion

We have investigated modulational instability of discrete dynamical structures in complex plasma system by considering damping (Epstein friction). We have shown that when damping contribution are not taken into account, the system reduces to the MDNLS equation. When the (Epstein) damping term, due to dust-neutral collisions is taken into account, the amplitude of oscillations is described by the MDCGL equation. We have found that modulational instability is related to localized patterns and the domain of instability increases with the damping due to dust-neutral collision and high values of damping extend the instability region. These localized excitations represent a general subset of large variety of possible news discrete solitons or already existing solutions admitted in our model. In forthcoming studies, for example, we should also investigate exact explicit soliton solution of our MDCGLS equation, to have some explanations of certain dust phenomenon in various states of the interstellar atmosphere.

Acknowledgements One of the authors (P. Dongmo) would like to thank Pr. J.P. Nguenang and Yurij Natanzon for discussions regarding numerical simulation, and Dr. G.L. Tiofack for his constructive suggestions. The authors also acknowledge the anonymous referees for their valuable suggestions.

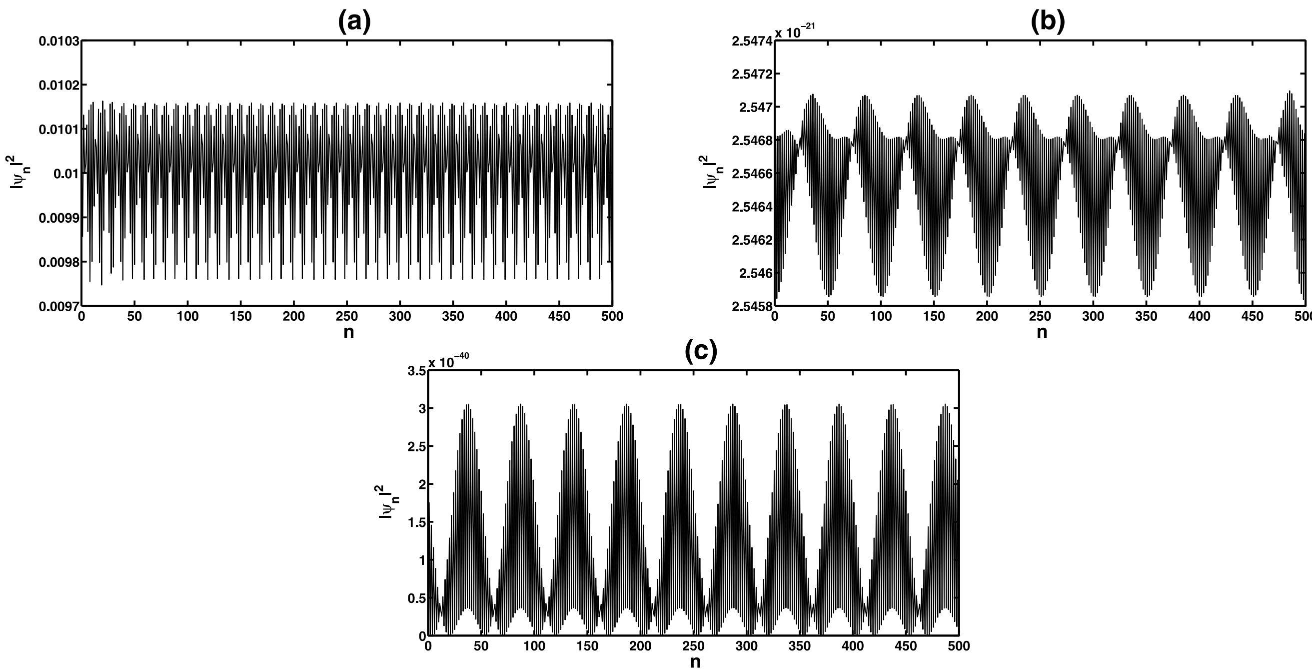

Fig. 4 Pattern formation. The amplitude appears as a train of soliton-like objects. Same parameters as in Fig. 1, for the carrier wave and modulated wave number (**a**): $q = 0.4\pi$, $Q = 0.9\pi$, $\nu = 0$; (**b**): $q = 0.98\pi$, $Q = 0.98\pi$, $\nu = 0.04$; (**c**): $q = 0.1\pi$, $Q = 0.98\pi$, $\nu = 0.075$

Appendix

The matrix elements m_{ij} $(i, j = 1, 2, 3, 4)$ are given as follows:

$$\begin{aligned}
m_1 &= \big(-4\cos(Q+2q) + 6\cos(q+Q) - 4\cos(q) \\
&\quad - 2\cos(Q-q) + 4\cos(2q)\big) R_i \psi_0{}^2 \\
&\quad + 2\big(\cos(q) - \cos(q+Q)\big) P_i, \\
m_2 &= \big(S_r + \big(-8\cos(q+Q) - 2\cos(2q) + 4\cos(Q) \\
&\quad + 4\cos(Q+2q)\big) R_r\big) \psi_0{}^2 \\
&\quad + \big(2\cos(q+Q) - 2\cos(q)\big) P_r, \\
m_4 &= -m_{10} = \big(\big(4\cos(Q) - 4\cos(q) - 2\cos(Q-q) \\
&\quad - 2\cos(q+Q) + 2\cos(2q)\big) R_r + S_r\big) \psi_0{}^2, \\
m_5 &= \big(2\cos(Q-q) + 4\cos(Q+2q) - 4\cos(2q) \\
&\quad - 6\cos(q+Q) + 4\cos(q)\big) R_r \psi_0{}^2 \\
&\quad + 2\big(\cos(q+Q) - \cos(q)\big) P_r, \\
m_6 &= \big(\big(-8\cos(q+Q) - 2\cos(2q) + 4\cos(Q) \\
&\quad + 4\cos(Q+2q)\big) R_i + S_i\big) \psi_0{}^2 \\
&\quad + \big(2\cos(q+Q) - 2\cos(q)\big) P_i, \\
m_8 &= -m_{14} = \big(\big(4\cos(Q) - 4\cos(q) - 2\cos(Q-q) \\
&\quad - 2\cos(q+Q) + 2\cos(2q)\big) R_i + S_i\big) \psi_0{}^2, \\
m_{11} &= -\big(-2\cos(q+Q) - 4\cos(q) + 6\cos(Q-q) \\
&\quad + 4\cos(2q) - 4\cos(Q-2q)\big) R_i \psi_0{}^2 \\
&\quad - 2\big(\cos(q) - \cos(Q-q)\big) P_i, \\
m_{12} &= -\big(S_r + \big(4\cos(Q-2q) + 4\cos(Q) - 8\cos(Q-q) \\
&\quad - 2\cos(2q)\big) R_r\big) \psi_0{}^2 - 2\big(\cos(Q-q) - \cos(q)\big) P_r, \\
m_{15} &= \big(-2\cos(q+Q) - 4\cos(q) + 6\cos(Q-q) \\
&\quad + 4\cos(2q) - 4\cos(Q-2q)\big) R_r \psi_0{}^2 \\
&\quad + 2\big(\cos(q) - \cos(Q-q)\big) P_r, \\
m_{16} &= \big(\big(8\cos(Q-q) + 2\cos(2q) - 4\cos(Q-2q) \\
&\quad - 4\cos(Q)\big) R_i - S_i\big) \psi_0{}^2 \\
&\quad + 2\big(\cos(q) - \cos(Q-q)\big) P_i; \\
K_3 &= -m_{15} - m_5 - m_2 - m_{12}, \\
K_2 &= m_4^2 + m_2 m_{12} + m_5 m_{15} - m_1 m_6 + m_5 m_2 + m_2 m_{15} \\
&\quad - m_{11} m_{16} + m_{12} m_{15} + m_5 m_{12}, \\
K_1 &= m_1 m_4 m_8 - m_5 m_2 m_{12} + m_1 m_6 m_{15} + m_5 m_{11} m_{16} \\
&\quad - m_5 m_2 m_{15} + m_2 m_{11} m_{16} + m_8 m_{11} m_4 - m_5 m_4{}^2 \\
&\quad - m_2 m_{12} m_{15} - m_4{}^2 m_{15} + m_1 m_6 m_{12} - m_5 m_{12} m_{15}, \\
K_0 &= -m_1 m_4 m_8 m_{15} + m_5 m_2 m_{12} m_{15} - m_5 m_2 m_{11} m_{16} \\
&\quad - m_1 m_6 m_{12} m_{15} + m_1 m_6 m_{11} m_{16} - m_5 m_8 m_{11} m_4 \\
&\quad + m_1 m_8{}^2 m_{11} + m_5 m_4{}^2 m_{15}.
\end{aligned}$$

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