Author copy.
Original published in: Journal of Plasma Physics, 82 (5) 905820508, (2016);
DOI: 10.1017/S0022377816000891.

Comment on *Weakly dissipative dust-ion acoustic wave modulation* [J. Plasma Phys. 82, 905820104 (2016)]

I. Kourakis[1] and I. S. Elkamash[2,1]

[1]*Centre for Plasma Physics, Queen's University Belfast, BT7 1NN Northern Ireland, UK*
[2] *Physics Department, Faculty of Science, Mansoura University, 35516 Mansoura, Egypt*

(Dated: October 12, 2016)

In a recent article [J. Plasma Phys. **82**, 905820104 (2009)], weakly dissipative dust-ion acoustic wave modulation in dusty plasmas was considered. It is shown in this Comment that the analysis therein involved severe fallacies, and is in fact based on an erroneous plasma fluid model, which fails to satisfy an equilibrium conditions, among other shortcomings. The subsequent analysis therefore is dubious and of limited scientific value.

In a recent article (Alinejad *et al* 2016) (henceforth to be referred to as *Paper 1*) an investigation was undertaken of the mechanism of weakly dissipative dust-ion acoustic wave modulation in dusty plasmas. It will be shown in the following that the analysis presented in that paper contains a number of intrinsic flaws, which renders the results of doubtful value.

The authors of Paper 1 consider the following fluid model:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = -\nu_r n_i + \nu_i n_e \,,$$
$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} = -\frac{\sigma}{n_i}\frac{\partial n_i}{\partial x} - \nu_e^{eff} u_i \,,$$
$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i + 1 - \mu \,; \qquad (1)$$

see Eqs. (2.1-3) in Paper 1. According to their formulation, the variables n_i, u_i and ϕ denote the ion number density, the ion fluid speed and the "electrostatic wave potential" (sic), respectively, while the electrons are taken to be Maxwellian, viz. their density reads $n_e = \mu e^{\phi}$. Note that the right-hand side (RHS) of the first (continuity) equation involves the parameters ν_r and ν_i, which allegedly represent the *"frequency of ion recombination on dust particles"* and the *"plasma ionization frequency"*, respectively, while the RHS in the second (momentum) equation involves the parameter ν_e^{eff} which denotes *"the effective frequency characterizing a loss in the ion momentum due to recombination on dust particles and Coulomb elastic collisions between ions and dust grains"* (quoting the authors in Paper 1). The dust component is explicitly assumed to be *"stationary"* (i.e., implied to be characterized by constant charge state Z_d and fixed number density n_d), and contributes to the model via the parameter $\mu = n_{e0}/n_{i0} = 1 - Z_d n_{d0}/n_{i0}$.

A number of remarks and comments are in row.

Comment 1. Clearly, the total number of the ions is *not conserved* within the aforementioned model: this is reflected by the non-zero RHS in the first (ion continuity) equation. Physically speaking, it is implied that both ions and electrons populate the dust grains dynamically, as suggested by the charging rates ν_r and ν_i (defined above). However, if this was true, the charge state Z_d would -obviously- not remain constant, as implied in the model, nor would the total electron density (implicitly assumed to be equal to $n_e = \mu$, i.e. constant).

It is evident that the model (along with the accompanying parameter definitions) has been inspired from a number of previous works (Goree *et al.* 1999; Vladimirov, Ostrikov & Yu 1999; Cramer & Vladimirov 2001; Popel *et al.* 2003), which are explicitly cited in Paper 1. However, it must be emphasized that the models proposed in those articles consider a *variable* dust charge *and* dust density, and are thus consistent (in contrast with Paper 1), both physically and mathematically. In particular, the model employed by Vladimirov, Ostrikov & Yu (1999) involves variable electron *and* ion number density [see eqs. (1) and (3) in the latter reference], but this is done in conjunction with variable dust number density and dust charge dynamics [note eqs. (4-5) in the same reference]. That model may not be applied gratuitously, if one considers a constant dust charge, stationary dust grains and inertialess electrons, as it the case in Paper 1.

From a fundamental point of view, it appears that Alinejad *et al* (Alinejad *et al* 2016) have taken into account the effects of ion recombination on the dust particles, and that of ionic momentum loss due to recombination on the dust particles and also due to electrostatic collisions between ions and dust grains, but they have neglected the effect of dust charging. The frequency scales characterizing these effects are typically of the same order of magnitude (Mamun & Shukla, 2002), and therefore these should have been considered on equal footing in the model. This assumption can therefore not be justified physically, nor mathematically.

Concluding our first observation, the ion-fluid model introduced in Paper 1 is of limited validity and of questionable physical value, as it fails to preserve the ion number density, entailing a dubious and unclear physical interpretation of the associated results. Although dust charging implies precisely a variation of the electron and ion population density, this is not taking into account properly through the specific fluid model considered in Paper 1 (Alinejad *et al* 2016).

Comment 2. The authors of Paper 1 consider an *equilibrium state*, namely $S^{(0)}$ in their eq. (3.1), which is later defined as the triad (vector) $S^{(0)} = (n^{(0)}, u^{(0)}, \phi^{(0)}) = (1, 0, 0)$. This equilibrium state is then used as reference state, around which the state variables are expanded in a polynomial series in ϵ ($\ll 1$): cf. Eq. (3.1) in Paper 1.

A crucial point needs to be made at this stage. It is straightforward to see that the above reference state does *not* satisfy the system of fluid equations (1)! To see this, one may simply substitute for $(n^{(0)}, u^{(0)}, \phi^{(0)}) = (1, 0, 0)$ into (1), to find that the first of Eqs. (1) yields a non-zero RHS in this case. This is essentially an algebraic manifestation of the physical fact that (as mentioned above) ion continuity is not preserved through this model.

Interestingly, Alinejad *et al* overcome the latter issue (i.e., the lack of equilibrium state in their model) by inventing an analytical trick, namely by scaling down the (fictitious) ion and electron annihilation mechanisms to order ϵ^2. It turns out that the linear dispersion characteristics of the model, expressed at first and second order in this perturbation method (Kourakis and Shukla, 2005) are thus left unaffected by the erroneous physical mechanism introduced (i.e., ion and electron number variation). As a consequence, expressions (3.3-5) in Paper 1, providing the dispersion relation, the first-order amplitude corrections and the group velocity, coincide with earlier results Kourakis & Shukla (2003, 2004), as the authors of Paper 1 correctly point out, in corroboration of their result.

The procedure outlined in Paper 1 is then pursued in third order, leading to a dissipative nonlinear Schrödinger equation (NLSE) in the form:

$$i\frac{\partial\phi}{\partial\tau} + P\,\frac{\partial^2\phi}{\partial\xi^2} + Q|\phi|^2\phi = -iR\phi\,, \qquad (2)$$

where ϕ is now redefined as the first-order correction to the electrostatic potential, and ξ and τ are space and time coordinates, consistently defined in the process (Alinejad *et al*, 2016). Not unexpectedly, the (linear) dispersion coefficient P in the NLSE – see (3.12-33a) in Paper 1 – is left unaffected by the artificial dissipation mechanism discussed above. This is simply due to the fact that P is related to the linear dynamics of the problem. This brings us to a third important comment.

Comment 3. It is known that the dispersion coefficient in the NLSE (2) above *must* satisfy the relation $P = \frac{1}{2}\frac{d^2\omega}{dk^2}$. This is an explicit byproduct of the multiple scale technique adopted in Paper 1; see e.g. in Kako (1972), Kakutani & Sugimoto (1974) or in Kourakis & Shukla (2003, 2004, 2005). One is tempted to test whether this requirement is met, by combining expressions (3.3) and (3.13a) in Paper 1, for $\omega(k)$ and $P(k)$, respectively. It turns out, upon simple substitution and some straightforward algebra, that this relation is *not* satisfied (!), viz., $P \neq \frac{1}{2}\frac{d^2\omega}{dk^2}$ – referring to Eqs. (3.3) and (3.13a) in Paper 1, specifically. Eq. (3.13a) is therefore not correct, and the subsequent analysis is presumably wrong (algebraically speaking, hence physically too).

It should be stated, in passing, that the expression derived for the nonlinearity Q – see (3.13b) in Paper 1 – should normally coincide with the earlier result in the "dissipation-free" case (Kourakis & Shukla, 2004). This is implied, but not rigorously shown not discussed in Paper 1.

By assuming that the RHS of the ion continuity equation scales as $\sim \epsilon^2$, the algebraic effect of the artificial dissipation mechanism thus introduced is "boosted" to order ϵ^3, and thus naturally appears (and is limited to) the damping term $-iR\phi$ appearing in the RHS of the NLSE, i.e. (3.12) in Paper 1. This builds up a straightforward and somehow "legitimate" algebraic model, with a rather not surprising outcome (a dissipative NLSE), yet with dubious physical interpretation, as discussed above.

Comment 4. It can actually be shown that a dissipative NLSE in the form of (2) above can be obtained by considering any linear combination of the state variables (i.e. terms of the form $\nu_1 n_i$, $\nu_2 u_i$ etc., adopting an *ad hoc* notation here) in the RHS of the evolution equation. Such a procedure defines an interesting algebraic procedure, in that it introduces a "dissipative fluid model" which can be analyzed as shown in Paper 1. It remains to see how realistic, and physically acceptable, this model is (see our discussion above).

According to the above considerations, one raises the question of the validity of the results in Paper 1. It may be argued that, focusing on the formal structure of the dissipative NLSE (2) above, and ignoring for a minute the definitions of the coefficients in it, that the analysis of modulational instability presented in Section 4 of Paper 1 is interesting and valuable *per se*. A twofold approach may be adopted here. First of all, a crucial point in Section 4 of Paper 1 is the derivation of the expression

$$\Delta(\tau) = -Q\phi_0^2 \exp(-2R\tau) \qquad (3)$$

for the nonlinear frequency shift [see the discussion following (4.1) in Paper 1], assuming an unperturbed amplitude (argument, in the polar representation) $\phi_0 = \mathrm{Arg}(\phi)$ (recall that ϕ is complex). Formally, this is tantamount to a transformation of the form $Q \to Q\exp(-2R\tau)$, which naturally entails an exponential decay of the critical wavenumber $K_c = (2Q/P)^{1/2}$, viz.

$$K_c = (2Q/P)^{1/2} \quad \to \quad (2Q/P)^{1/2}\exp(-R\tau)\,. \qquad (4)$$

The authors admit that this procedure is adapted from Xue (2003), who described dust-acoustic wave modulation taking into account dust-charge fluctuations [a different physical problem, nonetheless leading to an equation formally identical to (2) above]. It can be pointed out that the above result, as expressed, say, in relations (3) and (4) above, is rather trivial, as it follows directly from a simple transformation of the form

$$\phi \qquad \rightarrow \qquad \phi' \exp(-R\tau)\,. \tag{5}$$

In other words, it may be shown upon simple substitution of (5) into (2), that (2) becomes:

$$i\frac{\partial \phi'}{\partial \tau} + P\,\frac{\partial^2 \phi'}{\partial \xi^2} + Q'|\phi'|^2\phi' = 0\,, \tag{6}$$

where $Q' = Q\exp(-2R\tau)$. The analytical findings in the first part of Sec. 4 in Paper 1, relying on equations in the form of our Eqs. (3) and (4) above in particular (for the exponentially decaying nonlinear frequency shift Δ and for the wavenumber threshold K_c), thus simply follow from the above considerations, combined into the standard definitions for these quantities; see e.g. in Kourakis & Shukla (2003, 2004, 2005). For instance, the monochromatic wave solution of the "standard" form of the NLSE (6) reads $\phi' = |\phi'|\exp(-Q'|\phi'|^2\tau)$, which immediately yields the frequency shift $\Delta = -Q'|\phi'|^2 = -Qe^{-2R\tau}|\phi'|^2$, i.e. precisely Eq. (3) above (upon a trivial change in notation).

It follows that the plots presented in Figs. 2 and 3 in Paper 1 are founded on the above rationale, while depending on the actual definition of R - which is dubious, as discussed earlier. Regretfully, the graphical results presented in Paper 1 are therefore of doubtful value, as they reply on the (questionable) physical assumptions of the model, as discussed above.

The above suggests that the modulational instability related results in (Alinejad *et al* 2016) (namely, the instability growth and associated critical wavenumber) are not erroneous, but could have been obtained via a simpler analysis, than the one adopted – in turn based on Xue (2003).

In conclusion, we have shown that the fluid model presented in (Alinejad *et al* 2016) is intrinsically flawed, as it involves insurmountable errors in its physical interpretation, but also algebraic errors in the perturbative analysis presented therein. The results are therefore of no physical value. Admittedly, the modulational stability obtained in that article, based on a generic form of the dissipative NLS equation, is legitimate and perhaps interesting, but the outcome is rather trivial (as it can be recovered upon a simple phase-transformation from the original equation). Still, one is led to questionable results, once the so called ion-dust and electron-dust collisions are taken into account.

ALINEJAD, H., MAHDAVI M. & SHAHMANSOURI M. 2016 Weakly dissipative dust-ion acoustic wave modulation. J. Plasma Phys. **82**, 905820104 (2016).

CRAMER, N. F. & VLADIMIROV, S. V. 2001 Waves in dusty plasma discharges. Phys. Scr. T **89**, 122.

GOREE, J., MORFILL, G. E., TSYTOVICH, V. N. & VLADIMIROV, S. V. 1999 Theory of dust voids in plasmas. Phys. Rev. E **59**, 7055.

KAKUTANI, T. & SUGIMOTO, N. 1974 Krylov-Bogoliubov-Mitropolsky method for nonlinear wave modulation. Phys. Fluids, **17** (8), 1617.

KAKO, M. 1972 Nonlinear Wave Modulation in Cold Magnetized Plasmas. J. Phys. Soc. Japan, **33** (6), 1678.

KOURAKIS, I. & SHUKLA, P. K. 2003 Modulational instability and localized excitations of dust-ion acoustic waves. Phys. Plasmas **10**, 3459.

KOURAKIS, I. & SHUKLA, P. K. 2004 Finite ion temperature effects on oblique modulational stability and envelope excitations of dust-ion acoustic waves. Eur. Phys. J. D **28**, 109.

KOURAKIS, I. & SHUKLA, P.K. 2005 Exact theory for localized envelope modulated electrostatic wavepackets in space and dusty plasmas. Nonlin. Proc. Geophys., **12**, 407 (2005).

MAMUN, A.A & & SHUKLA, P.K. 2002 Introduction to Dusty Plasma Physics. IOP Publishing Ltd (Bristol and Philadelphia).

POPEL, S. I., GOLUB, A. P., LOSSEVA, T. V., IVLEV, A. V., KHRAPAK, S. A. & MORFILL, G. 2003 Weakly dissipative dust-ion-acoustic solitons. Phys. Rev. E 67, 056402.

VLADIMIROV, S. V., OSTRIKOV, K. N. & YU, M. Y. 1999 Ion-acoustic waves in a dust-contaminated plasma. Phys. Rev. E **60**, 3257.

XUE, J. K. 2003 Modulation of dust acoustic waves with non-adiabatic dust charge fluctuations. Phys. Lett. A **320**, 226.