THE ASTROPHYSICAL JOURNAL, 548:269–277, 2001 February 10
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INSTABILITIES IN A NONUNIFORM PARTIALLY IONIZED ISOTHERMAL MAGNETOPLASMA

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Received 2000 May 4; accepted 2000 October 16

ABSTRACT

Electromagnetic modes and associated instabilities in a partially ionized, self-gravitating, nonuniform, isothermal, initially static magnetoplasma are theoretically investigated. The effects of the neutral fluid dynamics, the inhomogeneities in the external magnetic field and the background plasma number density, the self-gravitational field, the drag forces, the ionization and recombination, etc., on the dispersion properties of electromagnetic waves for different frequency regimes of interest are examined. It is shown that new types of instabilities, the first type of which is due to the combined effects of the self-gravitational and the drag forces, the second type of which is due to the combined effects of the plasma and the neutral fluid dynamics, and the third type of which is due to the drifting of electrons, exist in such a magnetoplasma. Furthermore, the effects of the external magnetic field and its inhomogeneity, the neutral fluid temperature, collisions of electrons or ions with neutrals, and the ionization play the stabilizing role, whereas the effects of the neutral fluid mass density, the plasma number density inhomogeneity, collisions of neutrals with electrons or ions, and the recombination play the destabilizing role. It is suggested that such instabilities could be responsible for the fragmentation of molecular clouds into substructures that in turn may fail to be supported against the self-gravity and may collapse to form the stars.

Subject headings: instabilities — ISM: clouds — ISM: molecules — MHD — plasmas

1. INTRODUCTION

Studies of waves and associated instabilities in partially ionized plasma (Stenflo 1985; Stenflo, Tsintsadze, & Buadze 1989; Pudritz 1990; Carlberg & Pudritz 1990; Huba 1991; Huba, Grossmann, & Ottinger 1994; Birk, Kopp, & Shukla 1996; Suzuki & Sakai 1996; Daughton et al. 1998; Zweibel 1998; Stenflo 1999), which is quite common in most space and astrophysical plasma systems, particularly in molecular clouds, protostellar disks, interstellar and circumstellar clouds, the ionosphere of the Earth, etc. (Huba 1990; Suzuki & Sakai 1996; Bulanov & Sakai 1998; Zweibel 1999), have received a remarkable renewed interest because of their vital role in understanding the dynamics and fragmentation of molecular clouds, star formation, galactic structure and its evolution, magnetic field reconnection, the evolution of the magnetic field in molecular clouds, etc. (Norman & Heyvaerts 1985; Myers & Goodman 1988; Zweibel 1989; Huba 1990, 1991; Huba & Fedder 1993; Suzuki & Sakai 1996; Bulanov & Sakai 1998; Zweibel 1998, 1999).

The waves and instabilities in such a partially ionized magnetoplasma can be studied in any of three possible regimes, namely, (1) the electromagnetic force is much stronger than the gravitational force, (2) the electromagnetic force is of the same order of magnitude as the gravitational force, and (3) the electromagnetic force is much weaker than the gravitational force. Case 1 corresponds to usual laboratory plasma situations where Coulombic interaction is primarily responsible for the plasma dielectric behavior. Case 2 corresponds to planetary atmospheres and interstellar media, where the thickness of the Jovian ring, the spoke formation in the Saturn rings, etc., are thought to be due to the balance of these two forces (Alfvén & Mendis 1983). Case 3 generally corresponds to astrophysical plasmas where the formation of large-scale structures is attributed to gravitational condensation (Binney & Tremaine 1987).

A number of investigations of waves and instabilities in a self-gravitating partially ionized magnetoplasma have been done in the last few years by a number of authors, for example, Pudritz (1990), Carlberg & Pudritz (1990), Suzuki & Sakai (1996), Zweibel (1998), etc. Pudritz (1990) performed a local linear instability analysis of a self-gravitating partially ionized magnetoplasma and reported the stability results for the molecular clouds. Carlberg & Pudritz (1990) studied the role of hydrodynamic waves in stabilizing molecular clouds using numerical simulations. Suzuki & Sakai (1996) investigated the propagation of electromagnetic Alfvén waves in a self-gravitating weakly ionized plasma and showed that the small-amplitude Alfvén waves are damped because of ambipolar diffusion (the process by which the magnetic field carried by the ions diffuses through the neutral gas). Recently, Zweibel (1998) did a Jeans analysis to study the magnetosonic mode in a weakly ionized, self-gravitating magnetoplasma and showed that the mode becomes unstable because of the ambipolar drift (of ions) caused by the perturbed magnetic field.

However, most of the earlier investigations of waves and associated instabilities in partially ionized, self-gravitating magnetoplasmas (where the density inhomogeneity, the ion polarization, and the plasma effects are common events and play a very important role) have failed to recognize the importance of the density inhomogeneity, the ion polarization, and the plasma effects, which were considered by Huba (1990; 1991) in connection with a magnetic drift mode. We note that the plasma effect in the Jeans instability has been incorporated by Balsara (1996) and Zweibel (1998) through the ambipolar diffusion arising from the balance of the ion-neutral drag and the ion Lorentz force, by ignoring the Hall force, the ion inertial, and the gravitational force in the ion momentum equation. Thus, in the present investigation, we have considered a partially ionized, self-gravitating, nonuniform magnetoplasma and studied the electromagnetic modes and associated instabilities by taking into account the dynamics of the neutral and the plasma on an equal footing.

We have shown here that new types of instabilities, the first type of which is due to the combined effects of the self-gravitational force and collisions, the second type of which is due to the combined effects of the neutral and plasma fluid dynamics, and the third type of which is due to the drifting of electrons, exist in such a nonuniform magnetoplasma. The numerical analysis of the general dispersion relation exhibits several new features including different types of new instabilities and roles of the neutral dynamics, collisions, inhomogeneities, the external magnetic field, ionization and recombination, etc., in stabilizing or destabilizing these instabilities. It is suggested that these instabilities could be responsible for the fragmentation of molecular clouds into substructures that in turn may fail to be supported against the self-gravity and may collapse to form the stars.

The manuscript is organized as follows. We present the governing equations in § 2. We derive the dispersion relation and investigate the dispersion properties of the electromagnetic modes and associated instabilities in § 3. Section 3.1 is concerned with these modes and instabilities for incompressible neutral fluid, whereas § 3.2 deals with the compressible neutral fluid. A brief discussion is finally presented in § 4.

2. GOVERNING EQUATIONS

We consider a partially ionized, nonuniform, self-gravitating, isothermal magnetoplasma whose constituents are the electron, the ion, and the neutral fluid. The plasma is assumed to be quasi-neutral, i.e., $N_e = N_i = N$, where N_e (N_i) is the electron (ion) number density. We also assume that the plasma is immersed in a nonuniform external magnetic field $\boldsymbol{B}_0 = \hat{z}B_0(x)$, where $\hat{z}$ is the unit vector along the z-axis. The basic equations governing the macroscopic state of such a partially ionized, self-gravitating, nonuniform magnetoplasma system are given by

$$\frac{\partial N}{\partial t} + \nabla \cdot (N\boldsymbol{u}_i) = SN_n N - \alpha N^2 , \tag{1}$$

$$\frac{\partial N_n}{\partial t} + \nabla \cdot (N_n \boldsymbol{u}_n) = -SN_n N + \alpha N^2 , \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u}_i \cdot \nabla\right)\boldsymbol{u}_i = \frac{e}{m_i}\left(\boldsymbol{E} + \frac{1}{c}\,\boldsymbol{u}_i \times \boldsymbol{B}\right) - \frac{\gamma_i T_i}{m_i N}\nabla N - \nabla\Psi_g - \nu_{ie}(\boldsymbol{u}_i - \boldsymbol{u}_e) - \nu_{in}(\boldsymbol{u}_i - \boldsymbol{u}_n) + SN_n\boldsymbol{u}_n - \alpha N\boldsymbol{u}_i , \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u}_n \cdot \nabla\right)\boldsymbol{u}_n = -\frac{\gamma_n T_n}{m_n N_n}\nabla N_n - \nabla\Psi_g - \nu_{ni}(\boldsymbol{u}_n - \boldsymbol{u}_i) - \nu_{ne}(\boldsymbol{u}_n - \boldsymbol{u}_e) - SN\boldsymbol{u}_n + \alpha\frac{N^2}{N_n}\boldsymbol{u}_i , \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u}_e \cdot \nabla\right)\boldsymbol{u}_e = -\frac{e}{m_e}\left(\boldsymbol{E} + \frac{1}{c}\,\boldsymbol{u}_e \times \boldsymbol{B}\right) - \frac{\gamma_e T_e}{m_e N}\nabla N - \nu_{ei}(\boldsymbol{u}_e - \boldsymbol{u}_i) - \nu_{en}(\boldsymbol{u}_e - \boldsymbol{u}_n) + SN_n\boldsymbol{u}_n - \alpha N\boldsymbol{u}_i , \tag{5}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi eN}{c}(\boldsymbol{u}_i - \boldsymbol{u}_e) , \tag{6}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} , \tag{7}$$

and

$$\nabla^2\Psi_g = 4\pi G(m_i N + m_n N_n) , \tag{8}$$

where $\boldsymbol{u}_e$, $\boldsymbol{u}_i$, and $\boldsymbol{u}_n$ are, respectively, the electron, the ion, and the neutral fluid velocity, m_e, m_i, and m_n are, respectively, the electron, the ion, and the neutral mass, S and α are, respectively, the ionization and recombination coefficients, ν_{ie} (ν_{in}) is the collision frequency of ions with electrons (neutrals), ν_{ei} (ν_{en}) is the collision frequency of electrons with ions (neutrals), ν_{ni} (ν_{ne}) is the collision frequency of neutrals with ions (electrons), T_i, T_e, and T_n are, respectively, the constant ion, electron, and neutral temperature expressed in units of energy, γ_i, γ_e, and γ_n are, respectively, the adiabatic index for ion, electron, and neutral fluid, $\boldsymbol{E}$, $\boldsymbol{B}$, and Ψ_g are the electric field, the magnetic field, and the self-gravitational potential, respectively, e is the magnitude of the electron charge, G is the universal gravitational constant, and c is the speed of light in vacuum. To include the contributions due to the ionization and recombination in equations (1)–(5), we assume that the ions and the electrons are born moving at the velocity (u_n) of the neutrals and die by recombining at the velocity (u_i) of ions (at the moment of recombination, electron and ion velocities are equal). It should be noted that (i) the effect of the self-gravitational force on the electrons in equation (5) and the contribution due to the displacement current in equation (6) are neglected, (ii) to write equations (3) and (6) ions are assumed to be single charged, and (iii) equations (3), (4), and (5) hold for $m_i = m_n$.

3. DISPERSION PROPERTIES

To study the properties of electromagnetic waves in a partially ionized, self-gravitating, nonuniform magnetoplasma, we assume that the equilibrium density and magnetic field gradients are along the x-direction and there is no plasma or neutral flow at equilibrium. We shall carry out a normal mode analysis and, thus, express our dependent variables in terms of their equilibrium and isothermal perturbed parts as $N = n_0 + n$, $N_n = N_0 + n_n$, and $\boldsymbol{B} = \hat{z}B_0 + \boldsymbol{B}_1$, where $n \ll n_0$ ($n_n \ll N_0$) is the perturbed plasma (neutral) number density and $B_1 \ll B_0$ is the perturbed magnetic field. Thus, our basic equations (1)–(8) are linearized as

$$\frac{\partial n}{\partial t} + \nabla \cdot (n_0\boldsymbol{u}_i) = SN_0 n + Sn_0 n_n - 2\alpha n_0 n , \tag{9}$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot (N_0\boldsymbol{u}_n) = -SN_0 n - Sn_0 n_n + 2\alpha n_0 n , \tag{10}$$

$$\frac{\partial \boldsymbol{u}_i}{\partial t} = \frac{e}{m_i}\left(\boldsymbol{E} + \frac{1}{c}\,\boldsymbol{u}_i \times \boldsymbol{B}_0\right) - \frac{\gamma_i T_i}{m_i n_0}\nabla n - \nabla\Psi_g - \nu_{ie}(\boldsymbol{u}_i - \boldsymbol{u}_e) - \nu_{in}(\boldsymbol{u}_i - \boldsymbol{u}_n) + SN_0\boldsymbol{u}_n - \alpha n_0\boldsymbol{u}_i , \tag{11}$$

$$\frac{\partial \boldsymbol{u}_n}{\partial t} = -\frac{\gamma_n T_n}{m_n N_0}\nabla n_n - \nabla\Psi_g - \nu_{ni}(\boldsymbol{u}_n - \boldsymbol{u}_i) - \nu_{ne}(\boldsymbol{u}_n - \boldsymbol{u}_e) - S n_0 \boldsymbol{u}_n + \alpha \frac{n_0^2}{N_0}\boldsymbol{u}_i , \quad (12)$$

$$\frac{\partial \boldsymbol{u}_e}{\partial t} = -\frac{e}{m_e}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{u}_e \times \boldsymbol{B}_0\right) - \frac{\gamma_e T_e}{m_e n_0}\nabla n - \nu_{ei}(\boldsymbol{u}_e - \boldsymbol{u}_i) - \nu_{en}(\boldsymbol{u}_e - \boldsymbol{u}_n) + S N_0 \boldsymbol{u}_n - \alpha n_0 \boldsymbol{u}_i , \quad (13)$$

$$\nabla \times \boldsymbol{B}_1 = \frac{4\pi e n_0}{c}(\boldsymbol{u}_i - \boldsymbol{u}_e) , \quad (14)$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}_1}{\partial t} , \quad (15)$$

and

$$\nabla^2 \Psi_g = 4\pi G(m_i n + m_n n_n) . \quad (16)$$

To derive the dispersion relation for electromagnetic waves, we first use equations (13) and (14) and express equations (11) and (12) as

$$\left(\frac{\partial}{\partial t} + \nu_{in} + \alpha n_0\right)\boldsymbol{u}_i = \frac{B_0}{4\pi n_0 m_i}(\nabla \times \boldsymbol{B}_1) \times \hat{z} - \frac{c_s^2}{n_0}\nabla n - \nabla\Psi_g + (\nu_{in} + S N_0)\boldsymbol{u}_n \quad (17)$$

and

$$\left(\frac{\partial}{\partial t} + \nu_n + S n_0\right)\boldsymbol{u}_n = -\frac{v_{tn}^2}{N_0}\nabla n_n - \nabla\Psi_g + \left(\nu_n + \alpha\frac{n_0^2}{N_0}\right)\boldsymbol{u}_i - \frac{\nu_{ne} c}{4\pi e n_0}(\nabla \times \boldsymbol{B}_1) , \quad (18)$$

where $c_s^2 = (\gamma_e T_e + \gamma_i T_i)/m_i \simeq \gamma_e T_e/m_i$, $v_{tn}^2 = \gamma_n T_n/m_n$, and $\nu_n = \nu_{ni} + \nu_{ne}$. It may be mentioned that to derive equation (17) we have used $u_i + (m_e/m_i)u_e \simeq u_i$, $1 + m_e/m_i \simeq 1$, $\nu_{en} \ll \omega_{ce}$ (where $\omega_{ce} = eB_0/m_e c$ is the electron cyclotron frequency), and $\nu_{in} + (m_e/m_i)\nu_{en} \simeq \nu_{in}$ [since $\nu_{en} \simeq \nu_{in}(m_i/m_e)^{1/2}$]. Equations (17) and (18) are coupled with the wave magnetic field $\boldsymbol{B}_1$, the time evolution of which, after using equations (13), (14), and (15), is given by

$$\frac{\partial \boldsymbol{B}_1}{\partial t} = \nabla \times (\boldsymbol{u}_i \times \boldsymbol{B}_0) - \frac{c}{4\pi e}\nabla \times \left[\left(\frac{\nabla \times \boldsymbol{B}_1}{n_0}\right) \times \boldsymbol{B}_0\right] - \eta n_0 \nabla \times \left(\frac{\nabla \times \boldsymbol{B}_1}{n_0}\right) + \frac{\gamma_e T_e c}{e}\nabla \times \left(\frac{\nabla n}{n_0}\right) + \frac{m_e c}{e}\nabla \times (\nu_{en} + \alpha n_0)\boldsymbol{u}_i - \frac{m_e c}{e}\nabla \times (\nu_{en} + S N_0)\boldsymbol{u}_n , \quad (19)$$

where $\eta = \lambda_e^2 \nu_e$, $\nu_e = \nu_{ei} + \nu_{en}$, $\lambda_e = c/\omega_{pe}$ is the electron skin depth, and $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency. We note that $|\partial/\partial t| \ll \omega_{ce}$ is also assumed.

Now, for simplicity of mathematics, which must not lose any physics insight, we assume that (1) the wave magnetic field $\boldsymbol{B}_1$ is directed along the z-axis (i.e., $\boldsymbol{B}_1 = \hat{z}B_1$), (2) all perturbed quantities vary along the y-axis, and (3) the wavelength of the perturbation mode is much smaller than the density or magnetic field inhomogeneity scale length. These assumptions along with equations (9) and (10) allow us to express equations (17), (18), and (19) as

$$\hat{\boldsymbol{O}} n = \frac{n_0}{N_0}\hat{\boldsymbol{P}} n_n + \frac{n_0}{B_0} V_A^2 \frac{\partial^2 B_1}{\partial y^2} , \quad (20)$$

$$\hat{\boldsymbol{Q}} n_n = \frac{N_0}{n_0}\hat{\boldsymbol{R}} n - \nu_{ne} V_n^* \frac{N_0}{B_0}\frac{\partial B_1}{\partial y} , \quad (21)$$

and

$$\left(\frac{\partial}{\partial t} + V_{nb}^* \frac{\partial}{\partial y} - \eta\frac{\partial^2}{\partial y^2}\right)B_1 = \frac{B_0}{n_0}\hat{\boldsymbol{M}} n - \frac{B_0}{n_0}\nu_I n_n , \quad (22)$$

where

$$\hat{\boldsymbol{O}} = \left(\frac{\partial}{\partial t} + \nu_{in} + \nu_R\right)\left(\frac{\partial}{\partial t} - \beta\nu_I + 2\nu_R\right) - (\nu_{in} + \beta\nu_I)\left(\nu_I - \frac{2}{\beta}\nu_R\right) - c_s^2\frac{\partial^2}{\partial y^2} - \omega_{Ji}^2 ,$$

$$\hat{\boldsymbol{P}} = \beta\nu_I\left(\frac{\partial}{\partial t} + \nu_{in} + \nu_R\right) + (\nu_{in} + \beta\nu_I)\left(\frac{\partial}{\partial t} + \nu_I\right) + \omega_{Jn}^2 ,$$

$$\hat{\boldsymbol{Q}} = \left(\frac{\partial}{\partial t} + \nu_n + \nu_I\right)\left(\frac{\partial}{\partial t} + \nu_I\right) + \nu_I(\nu_R + \beta\nu_I) - v_{tn}^2\frac{\partial^2}{\partial y^2} - \omega_{Jn}^2 ,$$

$$\hat{\boldsymbol{R}} = \left(\nu_n + \frac{1}{\beta}\nu_R\right)\left(\frac{\partial}{\partial t} - \beta\nu_I + 2\nu_R\right) - \left(\nu_I - \frac{2}{\beta}\nu_R\right)\left(\frac{\partial}{\partial t} + \nu_n + \nu_I\right) + \omega_{Ji}^2 ,$$

$$\hat{\boldsymbol{M}} = \frac{\partial}{\partial t} - \beta\nu_I + 2\nu_R - V_{De}\frac{\partial}{\partial y} ,$$

$\beta = N_0/n_0$, $\nu_I = S n_0$, $\nu_R = \alpha n_0$, $\omega_{Ji} = (4\pi G n_0 m_i)^{1/2}$, $\omega_{Jn} = (4\pi G N_0 m_n)^{1/2}$, $V_A = B_0/(4\pi n_0 m_i)^{1/2}$ is the Alfvén speed, $V_{De} = c_s\rho_s/L_n$, $\rho_s = c_s/\omega_{ci}$, $L_n = n_0/(\partial n_0/\partial x)$, $V_n^* = \omega_{ce}\lambda_e^2/L_n$, $V_{nb}^* = \omega_{ce}\lambda_e^2/L_{nb}$, $L_{nb}^{-1} = L_n^{-1} - L_b^{-1}$, and $L_b = B_0/(\partial B_0/\partial x)$. Equations (20), (21), and (22) are our desired three coupled equations, which can be reduced to a single one as

$$\left[(\hat{\boldsymbol{O}}\hat{\boldsymbol{Q}} - \hat{\boldsymbol{P}}\hat{\boldsymbol{R}})\left(\frac{\partial}{\partial t} + V_{nb}^*\frac{\partial}{\partial y} - \eta\frac{\partial^2}{\partial y^2}\right) - (\hat{\boldsymbol{Q}}\hat{\boldsymbol{M}} - \beta\nu_I\hat{\boldsymbol{R}})V_A^2\frac{\partial^2}{\partial y^2} + (\hat{\boldsymbol{M}}\hat{\boldsymbol{P}} - \beta\nu_I\hat{\boldsymbol{O}})\nu_{ne}V_n^*\frac{\partial}{\partial y}\right]B_z = 0 . \quad (23)$$

Now, performing the Fourier transformation, i.e., taking $\partial/\partial t \to -i\omega$ and $\partial/\partial y \to ik$ (where $\boldsymbol{k}$ is along the y-axis only) and assuming $\nu_{ne}V_n^* \ll V_A^2\,\partial/\partial y$, from equation (23) we can

obtain a general dispersion relation for electromagnetic waves as

$$\omega^2 + i(\nu_{in} - \beta\nu_I + 3\nu_R)\omega + \omega_{Ji}^2 - k^2c_s^2 - \frac{k^2V_A^2[\omega + kV_{De} - i(\beta\nu_I - 2\nu_R)]}{\omega - kV_{nb}^* + i\eta k^2} = \frac{\omega_{Ji}^2 - i(\nu_n - \nu_I + 3\nu_R/\beta)\omega}{\omega^2 + i(\nu_n + 2\nu_I)\omega + \omega_{Jn}^2 - k^2v_{tn}^2} \times \left[\omega_{Jn}^2 - i(\nu_{in} + 2\beta\nu_I)\omega + \frac{i\beta\nu_I k^2V_A^2}{\omega - kV_{nb}^* + i\eta k^2}\right]. \quad (24)$$

Equation (24) is our new dispersion relation for electromagnetic waves modified by the combined effects of the neutral fluid dynamics, the self-gravitational force, the inhomogeneity in the background plasma density and the external magnetic field, the collisions, the ionization and the recombination, etc. We now study the electromagnetic perturbation mode and associated instabilities for two situations, namely, for an incompressible neutral fluid and for a compressible neutral fluid.

3.1. *Incompressible Neutral Fluid*

The dispersion relation for electromagnetic waves in a partially ionized plasma with an incompressible neutral fluid, i.e., $\omega \gg kv_{tn}$ and $\nu_n \to 0$, can be expressed from equation (24) as

$$\omega^2 + i(\nu_{in} - \beta\nu_I + 3\nu_R)\omega + \omega_{Ji}^2 - k^2c_s^2 - \frac{k^2V_A^2[\omega + kV_{De} - i(\beta\nu_I - 2\nu_R)]}{\omega - kV_{nb}^* + i\eta k^2} = 0\,. \quad (25)$$

Equation (25) represents the dispersion relation for electromagnetic waves including the combined effects of the self-gravitational force, the inhomogeneities in the external magnetic field and the background plasma number density, collisions of electrons and ions with stationary neutrals, the ionization, the recombination, etc. To study different modes and associated instabilities analytically, we consider the following special cases. It should be mentioned that the assumptions that we will consider here are not valid for any partially ionized/collisional plasma system simultaneously. However, these assumptions will provide us a clear theoretical understanding of the roles of the effects of different drag forces, ionization, recombination, etc., on the waves and associated instabilities under consideration.

3.1.1. *Case 1: Collisionless Homogeneous Plasma*

The consideration of collisionless, self-gravitating, uniform magnetoplasma (i.e., V_{nb}^*, V_{De}, η, ν_{in}, ν_I, $\nu_R \to 0$) allows us to simplify the dispersion relation (25) as

$$\omega \simeq \sqrt{k^2(c_s^2 + V_A^2) - \omega_{Ji}^2}\,. \quad (26)$$

This is the simplest dispersion relation for the magnetosonic waves modified by the effect of the self-gravitational field. It is obvious that the mode is subjected to the Jeans instability if $(c_s^2 + V_A^2) < \omega_{Ji}^2/k^2$. It is also clear that the external magnetic field and the electron or ion pressure play the stabilizing role, whereas the self-gravitational effect plays the destabilizing role.

3.1.2. *Case 2: Collisional Homogeneous Plasma*

We consider the homogeneous collisional plasma and examine the electromagnetic perturbation mode for different frequency regimes of interest.

3.1.2.1. *Case 2a: ηk^2, ν_I, $\nu_R \ll \omega$ and $\nu_{in} \gg \omega$*

The approximations ηk^2, ν_I, $\nu_R \ll \omega$ and $\nu_{in} \gg \omega$ allow us to simplify the dispersion relation (25) as

$$\omega \simeq i\,\frac{k^2}{\nu_{in}}\left[\frac{\omega_{Ji}^2}{k^2} - (c_s^2 + V_A^2)\right]. \quad (27)$$

It is obvious from equation (27) that the mode has no real frequency, and for $(c_s^2 + V_A^2)$ we have a purely growing dissipative magnetic Jeans instability due to the combined effects of the self-gravitational field and the collisions of ions with neutrals. On the other hand, for $(c_s^2 + V_A^2) > \omega_{Ji}^2/k^2$ it becomes a purely damped mode because of the collisional effect. It is clear that the effect of collisions of the ions with the neutrals does not have any role in stabilizing or destabilizing the mode, but this effect decreases the growth or damping rate of the purely growing or damped mode.

3.1.2.2. *Case 2b: ν_I, $\nu_R \ll \omega$ and ηk^2, $\nu_{in} \gg \omega$*

We now consider the assumptions ν_I, $\nu_R \ll \omega$ and ηk^2, $\nu_{in} \gg \omega$. Thus, in such a situation, the dispersion relation (25) can be simplified as

$$\omega \simeq i\eta\,\frac{\omega_{Ji}^2 - k^2c_s^2}{V_A^2(1 + \eta\nu_{in}/V_A^2)}\,. \quad (28)$$

This means that under the condition $kc_s < \omega_{Ji}$, there exists a purely growing dissipative magnetic Jeans instability, which is due to the combined effects of collisions of the electrons with the neutrals or the ions and the self-gravitational field. It is clear that, for $kc_s \ll \omega_{Ji}$ and $\eta\nu_{in} \ll V_A^2$, the growth rate of this purely growing instability is directly proportional to the collision frequency ν_e and to the ion-mass density ($n_0 m_i$) but is inversely proportional to the external magnetic field (B_0). On the other hand, for $kc_s \ll \omega_{Ji}$ and $\eta\nu_{in} \gg V_A^2$, the growth rate is almost independent of B_0 and ν_e, but is directly proportional to ω_{Ji}^2 and is inversely proportional to ν_{in}.

3.1.2.3. *Case 2c: ν_I, ν_R, ηk^2, $\nu_{in} \gg \omega$*

When we consider the assumptions ν_I, ν_R, ηk^2, $\nu_{in} \gg \omega$, the dispersion relation (25) can be simplified as

$$\omega \simeq i\,\frac{\omega_{Ji}^2 + (V_A^2/\eta)(\beta\nu_I - 2\nu_R) - k^2c_s^2}{\nu_{in} - \beta\nu_I + 3\nu_R}\,. \quad (29)$$

It is obvious from equation (29) that for $[\omega_{Ji}^2 + (V_A^2/\eta)(\beta\nu_I - 2\nu_R)] > k^2c_s^2$ and $\nu_{in} > (\beta\nu_I - 3\nu_R)$ we have a purely growing dissipative magnetic Jeans instability due to the combined effects of the self-gravitational field and the collisions of ions with neutrals or the ionization. It also provides us an interesting result that the combined effect of the external magnetic field and the ionization plays a destabilizing role for $\nu_{in} > (\beta\nu_I - 3\nu_R)$. This means that in this case the effect of the external magnetic field plays an opposite role to the one it plays in cases 2*a* and 2*b*. It is also seen that for $\nu_{in} \gg (\beta\nu_I - 3\nu_R)$ the effect of the ionization plays a destabilizing role, whereas that of the recombination plays a stabilizing role.

3.1.3. *Case 3: Collisional Inhomogeneous Plasma*

We introduce the collisional inhomogeneous plasma and study the electromagnetic perturbation mode for different frequency regimes of interest.

3.1.3.1. *Case 3a: ν_I, ν_R, kV_{De}, $\eta k^2 \ll \omega$ and kV_{nb}^*, $\nu_{in} \gg \omega$*

We assume the approximations ν_I, ν_R, kV_{De}, $\eta k^2 \ll \omega$ and kV_{nb}^*, $\nu_{in} \gg \omega$ and express the general dispersion relation

(25) as

$$\omega \simeq -kV_{nb}^{*}\,\frac{\omega_{Ji}^2 - k^2c_s^2}{k^2V_{\rm A}^2 + iv_{in}\,kV_{nb}^{*}}\,. \tag{30}$$

To examine whether this mode is stable or unstable or damped, we substitute $\omega = \omega_r + i\omega_i$ into equation (30). This substitution allows us to express the real (ω_r) and imaginary (ω_i) parts of ω as

$$\omega_r \simeq -kV_{nb}^{*}\left(\frac{\omega_{Ji}^2 - k^2c_s^2}{k^2V_{\rm A}^2 + v_{in}^2\,V_{nb}^{*2}/V_{\rm A}^2}\right) \tag{31}$$

and

$$\omega_i \simeq v_{in}\,\frac{V_{nb}^{*2}}{V_{\rm A}^2}\left(\frac{\omega_{Ji}^2 - k^2c_s^2}{k^2V_{\rm A}^2 + v_{in}^2\,V_{n}^{*2}/V_{\rm A}^2}\right). \tag{32}$$

It is obvious from equations (31) and (32) that, for $kc_s < \omega_{Ji}$, the mode becomes unstable because of the combined effects of the self-gravitational field and collisions of the ions with the neutrals and that, for $kc_s > \omega_{Ji}$, this will be a damped mode because of the collisional effect. It is observed that the normalized growth or damping rate ($\omega_i/|\,\omega_r\,| \simeq v_{in}\,V_{nb}^{*}/kV_{\rm A}^2$) of this growing or damped mode is directly proportional to the collision frequency v_{in} but is inversely proportional to the external magnetic field (B_0) and the inhomogeneity scale length (L_{nb}).

3.1.3.2. *Case 3b: v_I, v_R, $kV_{De} \ll \omega$ and $v_{in}, |i\eta k^2 - kV_{nb}^{*}| \gg \omega$*

Here, we assume the approximations v_I, v_R, $kV_{De} \ll \omega$ and $v_{in}, |i\eta k^2 - kV_{nb}^{*}| \gg \omega$, which allow us to consider the effect of V_{nb}^{*} on the purely growing dissipative magnetic Jeans instability (discussed in case 2*b*) or the effect of η on the electromagnetic mode (discussed in case 3*a*). Using the approximations v_I, v_R, $kV_{De} \ll \omega$ and $v_{in}, |i\eta k^2 - kV_{nb}^{*}| \gg \omega$, we can simplify the dispersion relation (25) as

$$\omega \simeq i\,\frac{(\omega_{Ji}^2 - k^2c_s^2)(\eta k^2 + ikV_{nb}^{*})}{v_{in}(\eta k^2 + ikV_{nb}^{*}) + k^2V_{\rm A}^2}\,. \tag{33}$$

Equation (33) represents the dispersion relation for the purely growing dissipative magnetic Jeans instability (discussed in case 2*b*) modified by the plasma number density inhomogeneity. This also stands for the dispersion relation for the electromagnetic mode (discussed in 3*a*) modified by the effect of collisions of the electrons with the neutrals or the ions. To examine whether the mode is stable or unstable or damped, we express the real (ω_r) and imaginary (ω_i) parts of ω as

$$\omega_r \simeq -kV_{nb}^{*}\left[\frac{\omega_{Ji}^2 - k^2c_s^2}{k^2V_{\rm A}^2(1 + \eta v_{in}/V_{\rm A}^2)^2 + v_{in}^2\,V_{nb}^{*2}/V_{\rm A}^2}\right] \tag{34}$$

and

$$\omega_i \simeq \left[v_{in}\,\frac{V_{nb}^{*2}}{V_{\rm A}^2} + \eta k^2\left(1 + \frac{\eta v_{in}}{V_{\rm A}^2}\right)\right] \times \left[\frac{\omega_{Ji}^2 - k^2c_s^2}{k^2V_{\rm A}^2(1 + \eta v_{in}/V_{\rm A}^2)^2 + v_{in}^2\,V_{nb}^{*2}/V_{\rm A}^2}\right]. \tag{35}$$

It is clear from equations (34) and (35) that, under the condition $kc_s < \omega_{Ji}$, electromagnetic waves propagating in a collisional, nonuniform, self-gravitating magnetoplasma suffer the magnetic Jeans instability, which is due to the combined effect of the collisions and the self-gravitational field. The real frequency of this mode is due only to the inhomogeneity in the plasma number density or in the external magnetic field. It is also obvious from equations (34) and (35) that for $V_{nb}^{*} \ll \eta k$ the mode corresponds to that (with $\omega_r \simeq 0$) discussed in case 2*b* and for $V_{nb}^{*} \gg \eta k$ the mode corresponds to the unstable perturbation mode (with $|\,\omega_r\,| > 0$) discussed in case 3*a*. This means that the effect of the inhomogeneity in the plasma density or the external magnetic field does not only modify the dispersion properties of such electromagnetic mode but also introduces a real frequency in the latter.

3.1.3.3. *Case 3c: $v_{in}, |i\eta k^2 - kV_{nb}^{*}|, v_I, v_R, kV_{De} \gg \omega$*

When we assume the approximations $v_{in}, |i\eta k^2 - kV_{nb}^{*}|, v_I, v_R, kV_{De} \gg \omega$, we can simplify the dispersion relation (25) and, as before, we can express the real (ω_r) and imaginary (ω_i) parts of ω for this case as

$$\omega_r \simeq \left(\frac{V_{\rm A}^2}{\eta^2k^2 + V_{nb}^{*2}}\right)\left[\frac{kV_{nb}^{*}(\beta v_I - 2v_R) + \eta k^3V_{De}}{v_{in} - \beta v_I + 3v_R}\right] \tag{36}$$

and

$$\omega_i \simeq \frac{1}{v_{in} - \beta v_I + 3v_R} \times \left\{\omega_{Ji}^2 - k^2c_s^2 + \frac{k^2V_{\rm A}^2}{\eta^2k^2 + V_{nb}^{*2}} \times [V_{De}\,V_{nb}^{*} - \eta(\beta v_I - 2v_R)]\right\}. \tag{37}$$

It is clear from equations (36) and (37) that for $\omega_{Ji}^2 + [k^2V_{\rm A}^2/(\eta^2k^2 + V_{nb}^{*2})][V_{De}\,V_{nb}^{*} - \eta(\beta v_I - 2v_R)] > k^2c_s^2$ and $v_{in} > (\beta v_I - 3v_R)$ the mode suffers the magnetic Jeans instability because of the combined effects of the self-gravitational field and collisions of the ions with the neutrals or the ionization. It also provides us with the interesting information that the combined effects of the external magnetic field and the inhomogeneities play a destabilizing role for kV_{De}, kV_{nb}^{*}, $v_{in} > (\beta v_I - 3v_R)$ and V_{De}, $V_{nb}^{*} \gg \eta k$. It is also seen that for $v_{in} > (\beta v_I - 3v_R)$ the effect of the recombination plays a destabilizing role whereas that of the ionization plays a stabilizing role.

3.2. *Compressible Neutral Fluid*

We are now interested in considering the compressible neutral fluid (i.e., $n_n \neq 0$ and $u_n \neq 0$), which allows us to study how the dynamics of the neutral fluid modifies the electromagnetic modes and associated instabilities discussed in the case of an incompressible neutral fluid. The dispersion relation for the electromagnetic perturbation mode in this general situation is given by equation (24). To investigate different modes and associated instabilities modified by the dynamics of the neutral fluid, we again consider the two different situations, namely, a collisionless homogeneous plasma and a collisional inhomogeneous plasma.

3.2.1. *Case 1: Collisionless Homogeneous Plasma*

We consider collisionless, self-gravitating, uniform magnetoplasma; i.e., we assume V_{nb}^{*}, V_{De}, η, v_{in}, v_n, v_I, $v_R \to 0$. The dispersion relation (24) can be simplified for this case as

$$\omega^2 \simeq \frac{1}{2}\,(k^2V_{s\rm A}^2 + k^2v_{tn}^2 - \omega_J^2) \times \left[1 \mp \sqrt{1 + 4\,\frac{\omega_{Jn}^2k^2V_{s\rm A}^2 + k^2v_{tn}^2(\omega_{Ji}^2 - k^2V_{s\rm A}^2)}{(k^2V_{s\rm A}^2 + k^2v_{tn}^2 - \omega_J^2)^2}}\right], \tag{38}$$

where $V_{sA}^2 = c_s^2 + V_A^2$ and $\omega_J^2 = \omega_{Ji}^2 + \omega_{Jn}^2$. Equation (38) describes the wave spectra that are due to the coupling of neutrals with the plasma via self-gravitation. If we neglect the plasma effect, we just obtain the dispersion relation for the sound wave in a self-gravitating neutral fluid ($\omega^2 = k^2 v_{tn}^2 - \omega_{Jn}^2$). On the other hand, if we consider cold and low-density neutral fluid, we obtain the dispersion relation for the magnetosonic mode discussed in case 1 of the incompressible neutral fluid. To see the effects of the cold neutral fluid on this magnetosonic mode we consider $|\omega_J^2 - k^2 V_{sA}^2| \gg \omega_{Jn} k V_{sA}$, which allows us to express the two solutions of equation (38) as

$$\omega^2 \simeq (k^2 V_{sA}^2 - \omega_J^2)\left[1 + \frac{\omega_{Jn}^2 k^2 V_{sA}^2}{(k^2 V_{sA}^2 - \omega_J^2)^2}\right] \quad (39)$$

and

$$\omega^2 \simeq \frac{\omega_{Jn}^2 k^2 V_{sA}^2}{\omega_J^2 - k^2 V_{sA}^2}. \quad (40)$$

It is shown from equations (39) and (40) that the destabilization of the short-wavelength (in comparison with the Jeans critical wavelength λ_c) mode is attributed to the last term of the square root of equation (39), which arises because of the coupling between the neutrals and the plasma. It is also seen that the combined effects of the plasma and the neutral dynamics do not only play an important role in destabilizing the magnetic Jeans instability or increasing its growth rate but also introduce a new mode approximately characterized by equation (40). It is seen that this mode disappears if the combined effect (or the effect of the neutral dynamics) is neglected. Equation (40) clearly indicates that, for $|\omega_J^2 - k^2 v_{sA}^2| \gg \omega_{Jn} k v_{sA}$, this new mode (which is due to the combined effect of the plasma and neutral dynamics) is unstable when the opposite Jeans criterion (i.e., $k^2 v_{sA}^2 > \omega_J^2$) is satisfied.

3.2.2. *Case 2: Collisional Inhomogeneous Plasma*

We have finally considered the collisional inhomogeneous plasma and studied numerically how the effects of the neutral fluid dynamics, the inhomogeneities in the background plasma density and the external magnetic field, the collisions, the ionization, the recombinations, etc., modify the dispersion properties of electromagnetic waves. For our numerical analysis of the general dispersion (24), we have chosen the plasma parameters that are typical for a number of space and astrophysical plasma systems, particularly for molecular clouds, protostellar disks, interstellar and circumstellar clouds, the ionosphere of the Earth, etc. (Huba 1990; Suzuki & Sakai 1996; Bulanov & Sakai 1998; Zweibel 1999): $\omega_{Jn}/\omega_{Ji} = 0$–$1.2$, $V_A/c_s = 0$–1.0, $v_{tn}/c_s = 0$–0.2, $V_{nb}^*/c_s = 0$–1.0, $V_{De}/c_s = 0$–0.7, $\nu_{in}/\omega_{Ji} = 0$–2.0, $\nu_{en}/\omega_{Ji} = 0$–0.5, $\eta\omega_{Ji}/c_s^2 = 0$–$0.5$, $\nu_I/\omega_{Ji} = 0$–0.6, and $\nu_R/\omega_{Ji} = 0$–0.6.

The advantage of choosing such normalized parameters is that our analysis can be valid for different ranges of some plasma parameters determining ω_{Ji} and c_s. As illustration, if we first take $T_e \simeq 100$ K, $n_0 \simeq 10^6$ cm^{-3}, and $m_i = 1.67 \times 10^{-24}$ g, we then have $\omega_{Ji} \simeq 1.2 \times 10^{-4}$ s^{-1} and $c_s = 9.1 \times 10^4$ cm s^{-1}. Hence, the corresponding parameters, which stand for our numerical analysis, are $n_n \simeq 0 - 1.44 \times 10^6$ cm^{-3}, $B_0 = 0$–0.4 mG, $v_{tn} = 0$–1.82×10^4 cm s^{-1}, $V_{nb}^* = 0$–9.1×10^4 cm s^{-1}, $V_{De} = 0$–6.37×10^4 cm s^{-1}, $\nu_{in} = 0$–2.4×10^{-4} s^{-1}, $\nu_{en} = 0$–0.6×10^{-4} s^{-1}, $\eta = 0$–3.49×10^{13} cm^2 s^{-1}, $\nu_I = 0$–0.72×10^{-4} s^{-1}, and $\nu_R = 0$–0.72×10^{-4} s^{-1}. It is obvious that these parameters are within the ranges that are typical for a number of space and astrophysical plasma systems, particularly for molecular clouds, protostellar disks, interstellar and circumstellar clouds, the ionosphere of the Earth, etc. (Huba 1990; Suzuki & Sakai 1996; Bulanov & Sakai 1998; Zweibel 1999).

Furthermore, in our second example, if we take $n_0 \simeq 10^3$ cm^{-3}, keeping the other parameters unchanged, we have $\omega_{Ji} \simeq 3.7 \times 10^{-6}$ s^{-1} and $c_s = 9.1 \times 10^4$ cm s^{-1}. Thus, the corresponding parameters, which stand for our numerical analysis, are $n_n \simeq 0 - 1.44 \times 10^3$ cm^{-3}, $B_0 = 0$–6.5 μG, $v_{tn} = 0$–1.82×10^4 cm s^{-1}, $V_{nb}^* = 0$–9.1×10^4 cm s^{-1}, $V_{De} = 0$–6.37×10^4 cm s^{-1}, $\nu_{in} = 0$–7.4×10^{-6} s^{-1}, $\nu_{en} = 0$–1.85×10^{-6} s^{-1}, $\eta = 0$–1.1×10^{15} cm^2 s^{-1}, $\nu_I = 0$–2.22×10^{-6} s^{-1}, and $\nu_R = 0$–2.22×10^{-6} s^{-1}. It should be pointed out that the parameters $n_n \simeq 10^3 - 10^6$ cm^{-3}, $B_0 = 1$ μG–1 mG, and $T_n \simeq 10$ K, which are, of course, within the range of our parameters mentioned in both of our two examples, are for the giant molecular clouds (Zweibel 1999).

The numerical results are displayed in Figures 1–7. Figures 1 and 2 show how the effects of the neutral fluid mass density, the external magnetic field, and the neutral fluid temperature modify the dispersion properties of the electromagnetic wave. It is seen from Figure 1 that, as we increase the neutral fluid mass density (keeping the other parameters constant), the critical value of the wavelength (the minimum value of the wavelength for which the electro-

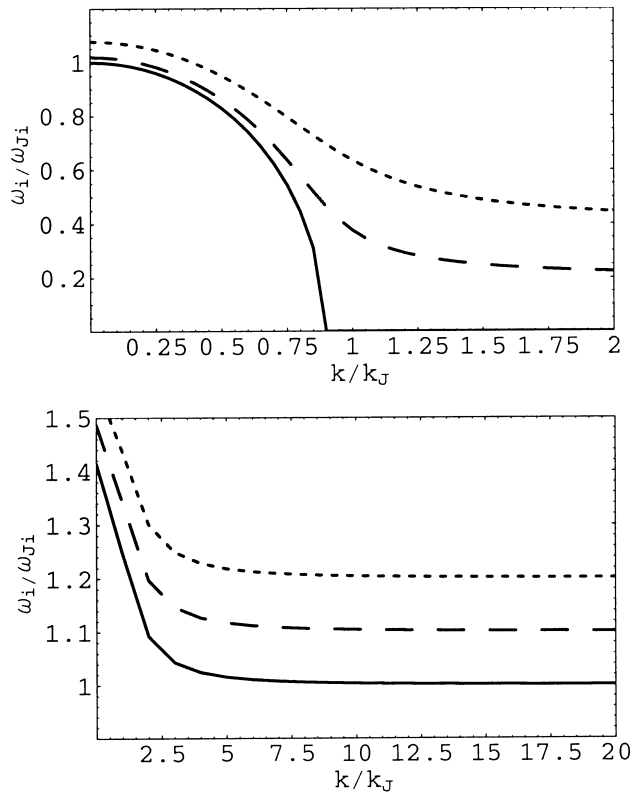

FIG. 1.—Effects of ω_{Jn} (*top*: $\omega_{Jn}/\omega_{Ji} = 0$, *solid curve*; $\omega_{Jn}/\omega_{Ji} = 0.2$, *dashed curve*; and $\omega_{Jn}/\omega_{Ji} = 0.4$, *dotted curve*; *bottom*: $\omega_{Jn}/\omega_{Ji} = 1.0$, *solid curve*; $\omega_{Jn}/\omega_{Ji} = 1.1$, *dashed curve*; and $\omega_{Jn}/\omega_{Ji} = 1.2$, *dotted curve*) on the growth rate of the electromagnetic mode for $V_A/c_s = 0.5$, $v_{tn}/c_s = 0.01$, $\nu_{in}/\omega_{Ji} = 0.1$, $\nu_n/\omega_{Ji} = 0.1$, $\eta\omega_{Ji}/c_s^2 = 0.01$, $V_{De}/c_s = 0.01$, $V_{nb}^*/c_s = 0.01$, $\nu_I/\omega_{Ji} = 0.01$, and $\nu_R/\omega_{Ji} = 0.01$. The wavenumber k is normalized by $k_J = \omega_{Ji}/c_s$.

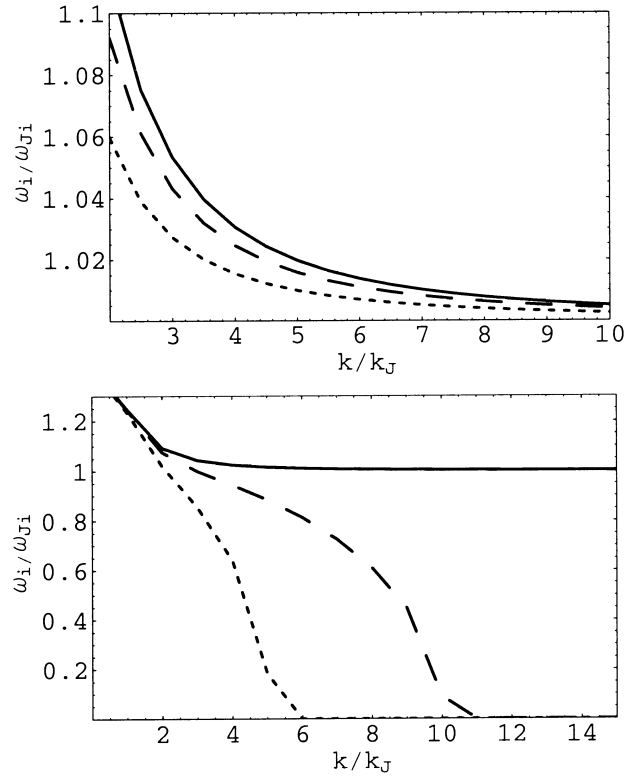

FIG. 2.—Effects of V_A (*top:* $V_A/c_s = 0$, *solid curve*; $V_A/c_s = 0.5$, *dashed curve*; and $V_A/c_s = 1.0$, *dotted curve*) and v_{tn} (*bottom:* $v_{tn}/c_s = 0$, *solid curve*; $v_{tn}/c_s = 0.1$, *dashed curve*; and $v_{tn}/c_s = 0.2$, *dotted curve*) on the growth rate of the electromagnetic mode for $\omega_{Jn}/\omega_{Ji} = 1.0$. The other parameters are the same as in Fig. 1.

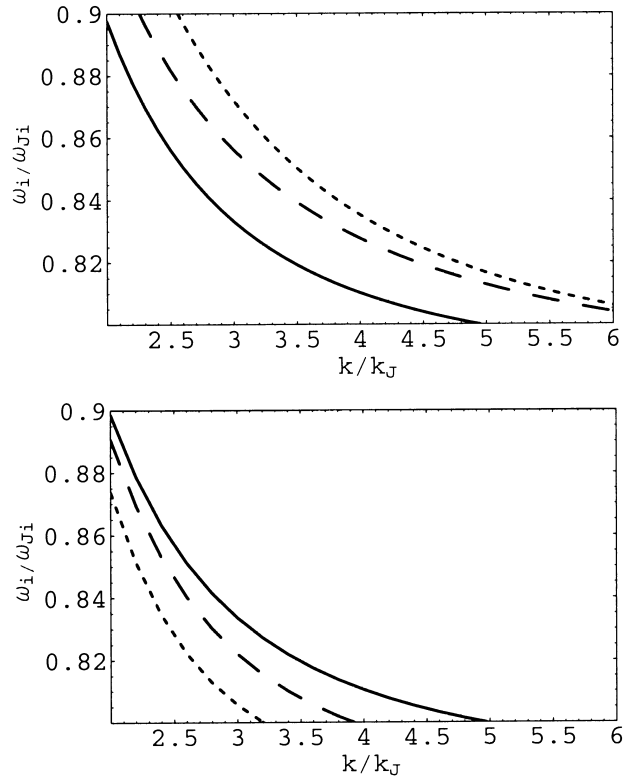

FIG. 3.—Effects of V^*_{nb} (*top:* $V^*_{nb}/c_s = 0$, *solid curve*; $V^*_{nb}/c_s = 0.5$, *dashed curve*; and $V^*_{nb}/c_s = 1.0$, *dotted curve*) and V_{De} (*bottom:* $V_{De}/c_s = 0.1$, *solid curve*; $V_{De}/c_s = 0.3$, *dashed curve*; and $V_{De}/c_s = 0.5$, *dotted curve*) on the growth rate of the electromagnetic mode for $\omega_{Jn}/\omega_{Ji} = 1.0$, $V_A/c_s = 1.0$, $v_{tn}/c_s = 0.01$, $v_{in}/\omega_{Ji} = 0.5$, $v_n/\omega_{Ji} = 0.1$, $\eta\omega_{Ji}/c_s^2 = 0.1$, $v_I/\omega_{Ji} = 0.01$, and $v_R/\omega_{Ji} = 0.01$.

magnetic perturbation suffers the magnetic Jeans instability) is shifted to lower values. Figure 2 indicates that the effect of the external magnetic field plays a stabilizing role (see upper panel of Fig. 2) whereas the effect of the neutral fluid temperature drastically changes the nature of the dispersion properties of the electromagnetic mode and tries to stabilize it (see lower panel of Fig. 2). Figures 3 and 4 show the effects of the inhomogeneities in the background plasma number density and the external magnetic field on the growth rate of the electromagnetic mode. It is found from Figure 3 that the plasma number density inhomogeneity plays the destabilizing role while the effects of the magnetic field inhomogeneity and drifting of the electrons play the stabilizing role. It is shown from Figure 4 that the drifting of electrons introduces another type of instability where the growth rate increases with the rise of the drift speed of electrons (see Fig. 4). Figures 5 and 6 show the effects of the ion-neutral collision frequency (v_{in}), the electron-neutral collision frequency (v_{en}), and the electron-ion collision frequency (v_{ei}) on the growth rate of the unstable electromagnetic mode. It is observed here that the growth rate decreases with v_{in} and v_{en} but increases with v_{ei}. Figure 7 shows how the effects of the ionization and the recombination modify the dispersion properties of the electromagnetic perturbation mode. It is obvious that the effect of the ionization plays the stabilizing role whereas the effect of the recombination plays the destabilizing role. It has been shown by our numerical analysis that, if we use the parameters mentioned in the first example, the growth rate and the corresponding wavelength of the unstable perturbation mode are, respectively, $\sim 10^{-4}$ s^{-1} and $\sim 10^9$ cm,

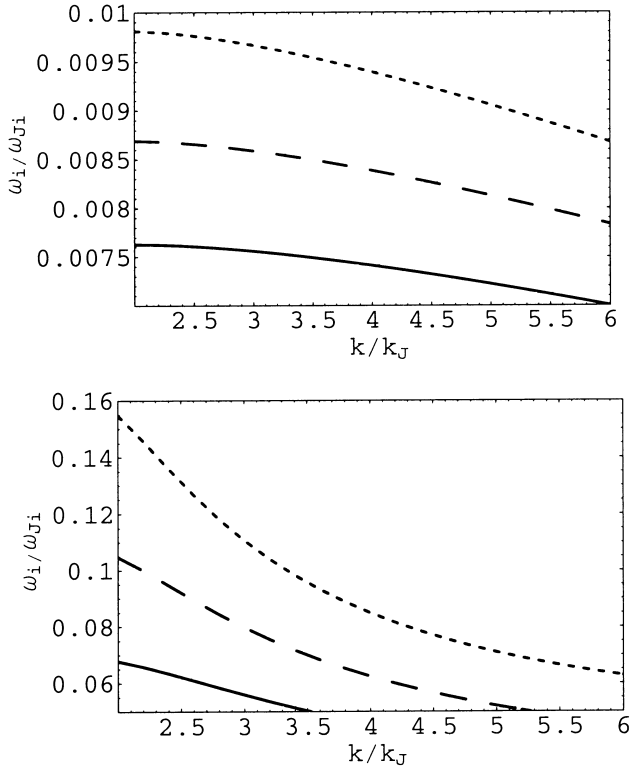

FIG. 4.—New unstable mode and variation of its growth rate with V_{De} (*top:* $V_{De}/c_s = 0.2$, *solid curve*; $V_{De}/c_s = 0.21$, *dashed curve*; and $V_{De}/c_s = 0.22$, *dotted curve*; *bottom:* $V_{De}/c_s = 0.5$, *solid curve*; $V_{De}/c_s = 0.6$, *dashed curve*; and $V_{De}/c_s = 0.7$, *dotted curve*) on the growth rate of the instability. The other parameters are the same as in Fig. 3.

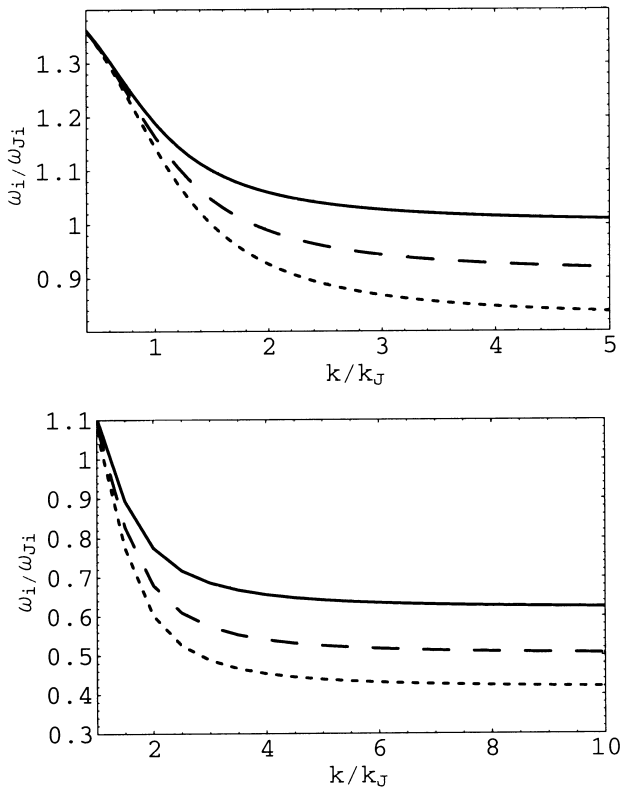

FIG. 5.—Effects of v_{in} (*top*: $v_{in}/\omega_{Ji} = 0$, *solid curve*; $v_{in}/\omega_{Ji} = 0.2$, *dashed curve*; and $v_{in}/\omega_{Ji} = 0.4$, *dotted curve*; *bottom*: $v_{in}/\omega_{Ji} = 1.0$, *solid curve*; $v_{in}/\omega_{Ji} = 1.5$, *dashed curve*; and $v_{in}/\omega_{Ji} = 2.0$, *dotted curve*) on the growth rate of the electromagnetic mode. The other parameters are the same as in Fig. 3.

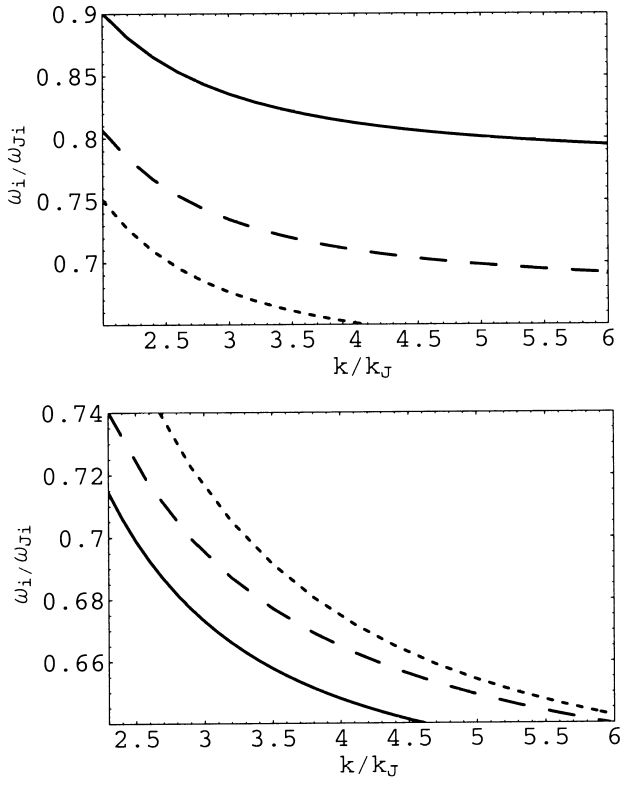

FIG. 6.—Effects of v_{en} (*top*: $v_{en}/\omega_{Ji} = 0$, *solid curve*; $v_{en}/\omega_{Ji} = 0.1$, *dashed curve*; and $v_{en}/\omega_{Ji} = 0.5$, *dotted curve*) and η (*bottom*: $\eta\omega_{Ji}/c_s^2 = 0$, *solid curve*; $\eta\omega_{Ji}/c_s^2 = 0.1$, *dashed curve*; and $\eta\omega_{Ji}/c_s^2 = 0.5$, *dotted curve*) on the growth rate of the instability. The other parameters are the same as in Fig. 3.

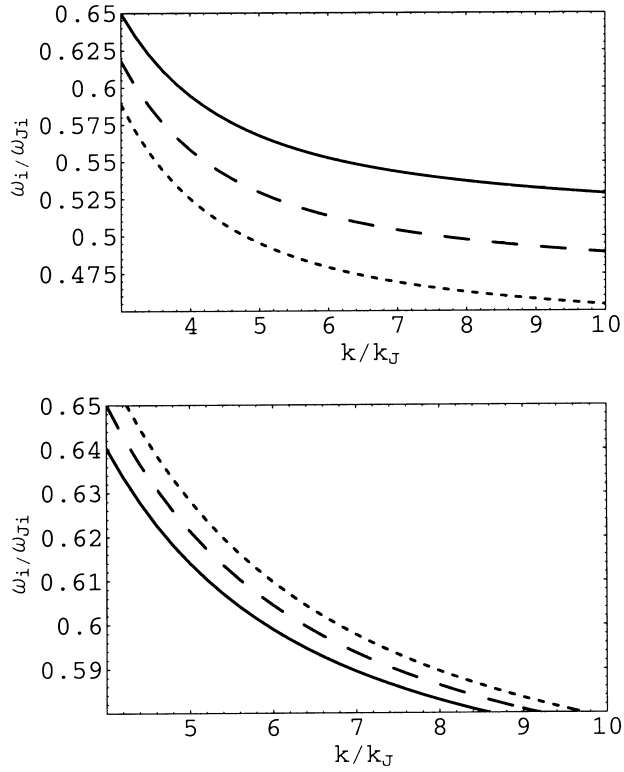

FIG. 7.—Effects of v_I (*top*: $v_I/\omega_{Ji} = 0.2$, *solid curve*; $v_I/\omega_{Ji} = 0.3$, *dashed curve*; and $v_I/\omega_{Ji} = 0.5$, *dotted curve*) and v_R (*bottom*: $v_R/\omega_{Ji} = 0.2$, *solid curve*; $v_R/\omega_{Ji} = 0.4$, *dashed curve*; and $v_R/\omega_{Ji} = 0.5$, *dotted curve*) on the growth rate of the magnetic Jeans instability.

whereas, if we use the parameters mentioned in the second example, the growth rate and the corresponding wavelength of the unstable perturbation mode are, respectively, $\sim 10^{-6}$ s^{-1} and $\sim 10^{11}$ cm.

4. DISCUSSION

We have presented a self-consistent description of electromagnetic waves and associated instabilities in a partially ionized, self-gravitating, nonuniform magnetoplasma, accounting for the neutral fluid dynamics, the drag forces, the ionization and recombination, the inhomogeneities in the background plasma number density and the external magnetic field, etc. We first derived a new dispersion relation (see eq. [24]) by employing the continuity and the momentum equations for the neutral fluid as well as the Hall magnetohydrodynamic (HMHD) equations for the plasma including the self-gravitational, the electromagnetic, and the drag forces. We then analytically studied the electromagnetic waves for different frequency regimes of interest. We also numerically analyzed the general dispersion relation and examined how the effects of the neutral fluid dynamics, the inhomogeneities in the background plasma number density and the external magnetic field, the drag forces, the ionization, the recombination, etc., modify the dispersion properties of the electromagnetic perturbation mode. The results that were found in this investigation may be summarized as follows.

1. A new growing dissipative magnetic Jeans instability, which is due to the combined effects of the self-gravitational force and collisions of ions with stationary neutrals or collisions of electrons with ions or stationary neutrals, exists in a self-gravitating, uniform magnetoplasma. It is observed

that, for $kc_s \ll \omega_{Ji}$ and $\eta v_{in} \ll V_A^2$, the growth rate of this purely growing instability is directly proportional to the collision frequency v_e but is inversely proportional to the external magnetic field (B_0). On the other hand, for $kc_s \ll \omega_{Ji}$ and $\eta v_{in} \gg V_A^2$, this growth rate is almost independent of B_0 and v_e but is inversely proportional to v_{in}.

2. The effect of the plasma density inhomogeneity on this dissipative magnetic Jeans instability does not only modify the dispersion properties of such electromagnetic waves but also introduces a real frequency. We have shown that, for $kc_s < \omega_{Ji}$, the electromagnetic mode becomes unstable because of the combined effects of the self-gravitational field and the collisions of ions or electrons with stationary neutrals and that, for $kc_s > \omega_{Ji}$, this will be a damped mode due to this collisional effect. It is observed that the normalized growth or damping rate of the growing or the damped mode is directly proportional to the collision frequency v_{in} but is inversely proportional to the inhomogeneity scale length (L_n).

3. The combined effects of the plasma and the neutral fluid dynamics do not only play an important role in destabilizing the magnetic Jeans instability or increasing its growth rate but also introduce a new mode (see eq. [40]). It is seen that this mode disappears if the combined effects (or the effect of the neutral dynamics) are neglected. It is also shown that, for $|\omega_J^2 - k^2 V_{sA}^2| \gg \omega_{Jn} k V_{sA}$, this new mode is unstable when the opposite Jeans criterion (i.e., $k^2 V_{sA}^2 > \omega_J^2$) is satisfied.

4. It is found that, as we increase the neutral fluid mass density (keeping the other parameters constant), the critical value of the wavelength (the minimum value of the wavelength for which the electromagnetic perturbation mode suffers magnetic Jeans instability) is shifted to lower values.

5. The effect of the external magnetic field plays a stabilizing role, whereas the effect of the neutral fluid temperature drastically changes the nature of the dispersion properties of the electromagnetic mode and tries to stabilize it.

6. The density inhomogeneity plays the destabilizing role, whereas the magnetic field inhomogeneity and the drifting of electrons play the stabilizing role. It is also shown that the drifting of electrons introduces another type of instability where the growth rate increases with the rise of the drift speed of electrons.

7. The growth rate of the unstable electromagnetic perturbation mode decreases with the ion-neutral collision frequency (v_{in}) and the electron-neutral collision frequency (v_{en}) but increases with the electron-ion collision frequency (v_{ei}). It is also shown that the effect of the ionization plays the stabilizing role whereas the effect of the recombination plays the destabilizing role.

It should be noted that the assumptions that we considered here are not valid for any partially ionized/collisional plasma system simultaneously. However, these assumptions provide us a clear theoretical understanding of the roles of the effects of different drag forces, ionization, recombination, etc., on the waves and associated instabilities under consideration.

It has been shown by our numerical analysis that, if we use the parameters mentioned in the first example, the growth rate and the corresponding wavelength of the unstable perturbation mode are, respectively, $\sim 10^{-4}$ s^{-1} and $\sim 10^9$ cm, whereas, if we use the parameters mentioned in the second example, the growth rate and the corresponding wavelength of the unstable perturbation mode are, respectively, $\sim 10^{-6}$ s^{-1} and $\sim 10^{11}$ cm.

It may be stressed here that the present investigation may be useful for understanding the propagation of electromagnetic perturbation and associated instabilities in dense molecular interstellar clouds and cometary plasmas that contain a significant fraction of neutral atoms and that are subjected to the self-gravitational force. It is suggested that the instabilities discussed here could be responsible for the fragmentation of molecular clouds into substructures that in turn may fail to be supported against the self-gravity and may collapse to form the stars in galaxies.

A. A. Mamun gratefully acknowledges the financial support of the Alexander von Humboldt-Stiftung (Bonn, Germany) and the study leave granted by the authority of Jahangirnagar University (Dhaka, Bangladesh).

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