Solitary potentials in cometary dusty plasmas

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1. Introduction

[2] The physics of charged dust particles, which are ubiquitous in space plasmas, has received much attention due to their crucial role in supporting electrostatic density perturbations and potential structures that are observed in different regions of space environments, namely lower and upper mesosphere, cometary tails, planetary rings, interplanetary spaces, interstellar media, etc. [Mendis and Rosenberg, 1994; Horányi and Mendis, 1986; Horányi, 1996; Mendis, 1997; Verheest, 2000; Shukla, 2001; Shukla and Mamun, 2002]. The collective processes (linear/nonlinear waves and associated instabilities) related to electrostatic density perturbations and potential structures observed in such different regions of space have been investigated based on a most commonly used dusty plasma system that contains negatively charged dust particles, ions, and electrons [Rao et al., 1990; Shukla and Silin, 1992; Mamun, 1999; Verheest, 2000; Shukla, 2001; Shukla and Mamun, 2002].

[3] The presence of positively charged dust particles have also been observed in different regions of space, viz. cometary tails [Mendis and Horányi, 1991; Chow et al., 1993; Mendis and Rosenberg, 1994; Horányi, 1996], Jupiter’s magnetosphere [Horányi et al., 1993], etc. There are three principal mechanisms by which a dust grain becomes positively charged [Fortov et al., 1998]. These are i) photo-emission in the presence of a flux of ultraviolet (UV) photons, ii) thermion emission induced by radiative heating, and iii) secondary emission of electrons from the surface of the dust grains.

[4] Chow et al. [1993] have theoretically shown that due to the size effect on secondary emission insulting dust grains with different sizes in space plasmas can have the opposite polarity (smaller ones being positive and larger ones being negative). This is mainly due to the fact that the excited secondary electrons have shorter (longer) distances to travel to reach the surface of the smaller (larger) dust grains (for details we refer to Chow et al. [1993]).

[5] Chow et al. [1993] have also calculated the equilibrium potential for insulting grains immersed in both Maxwellian and generalized Lorentzian plasmas. Due to the size effect on the secondary emission they have found that insulting grains with diameters 0.01 μm and 1 μm have opposite polarity (with smaller ones being positive) when the plasma temperature is in the range 25–48 eV for a Maxwellian plasma and in the range 7–17 eV for a Lorentzian plasma. These values may be in the range of the inferred values in different regions of planetary ring systems, comets, interplanetary medium, supernova remnants, etc. [Mendis and Horányi, 1991; Mendis and Rosenberg, 1994].

[6] There are also direct evidence for the existence of both positively and negatively charged dust particles in cometary tails [Mendis and Horányi, 1991; Mendis and Rosenberg, 1994; Horányi, 1996]. Motivated by the theoretical prediction [Chow et al., 1993] and experimental observations of [Mendis and Rosenberg, 1994; Mendis and Horányi, 1991; Horányi, 1996] opposite polarity dust grains in cometary tails, in this letter we have considered a new dusty plasma system, which contains positively and negatively charged dust particles (i.e., assumes a complete depletion of background electrons and ions), and have theoretically investigated the properties of linear and nonlinear electrostatic waves that may propagate in such a mesospheric dusty plasma.

2. Governing Equations

[7] We consider a two-component, uniform, mesospheric dusty plasma containing positively and negatively charged dust fluids. We assume that the negative dust particles are much more massive than positive ones [Mendis and Horányi, 1991; Chow et al., 1993; Mendis and Rosenberg, 1994; Horányi, 1996]. The basic governing equations are

\[
\frac{\partial n_1}{\partial t} + \frac{\partial}{\partial x}(n_1 u_1) = 0, \tag{1}
\]

\[
m \left( \frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} \right) = -\frac{\partial \Phi}{\partial x} \frac{1}{n_1} \frac{\partial p}{\partial x}, \tag{2}
\]

\[
\frac{\partial p}{\partial t} + u_1 \frac{\partial p}{\partial x} + 3p \frac{\partial u_1}{\partial x} = 0, \tag{3}
\]
for positive dust fluid dynamics and
\[
\frac{\partial n_2}{\partial t} + \frac{\partial}{\partial x}(n_2u_2) = 0, \quad (4)
\]
and for negative dust fluid dynamics. Here \( n_j \) is the particle number density normalized by its equilibrium value \( n_{0j} \) (\( j = 1 \) for the positively charged dust fluid and \( j = 2 \) for the negatively charged dust fluid), \( u_j \) is the fluid speed normalized by \( C_s = \sqrt{\mu V_T} \) \( [\mu = Z_jm_j/Z_jm_2, \ V_T = (k_BT/m_1)^{1/2}] \), \( Z_j \) \((Z_2)\) represents the number of the positive (negative) charges onto the positive (negative) dust grain surface, \( m_1 \) \((m_2)\) represents the mass of the positive (negative) dust particle, \( k_B \) is the Boltzmann constant, \( T \) is the temperature of the positive dust fluid, \( p \) is the thermal pressure of the positive dust fluid normalized by \( n_{10}k_B T \), \( \Phi \) is the wave potential normalized by \( k_B T Z_j e \), \( x \) is the space variable normalized by \( \lambda_{D1} = (k_B T Z_j/e)^{1/2} \), and \( t \) is the time variable normalized by \( \omega_{p2} = (m_2/4\pi n_{20}Z_2^2 e^2)^{1/2} \).

[8] Equations (1)–(5) are supplemented by Poisson’s equation
\[
\frac{\partial^2 \Phi}{\partial x^2} = n_2 - n_1. \quad (6)
\]

3. Linear Waves
[9] We first study the dispersion properties of the linear electrostatic waves, for which we express our dependent variables \( n_j, u_j, p, \text{and} \ \Phi \) in terms of their equilibrium and perturbed parts, viz. \( n_j = 1 + \delta n_j, \ u_j = u_0 + \delta u_j \) (we assume that at equilibrium there is no fluid flow), \( p = p_0 + \delta p \) and \( \Phi = \Phi_0 + \delta \Phi \) (we assume that at equilibrium there is no electric field), and substitute them into (1)–(6). We assume that all perturbed quantities are proportional to \( -i\Omega t + Kx \), where \( \Omega = \omega/\omega_{p2} \) with \( \omega \) being the wave frequency and \( K = k_{D1}l \) with \( k \) being the wave propagation constant. Thus, substituting \( \partial/\partial t = -i\Omega \) and \( \partial/\partial x = ik \) into the linearized form of (1)–(6), we have the linear dispersion relation
\[
\omega^2 = \frac{1}{2} \omega_{p1}^2 (1 + \mu + 3k^2\lambda_{D1}^2) \left[ 1 \pm \frac{12\mu k^2\lambda_{D1}^2}{(1 + \mu + 3k^2\lambda_{D1}^2)} \right].
\]
Equation (7) is the dispersion relation for two types of waves: one corresponding to \( + \) sign and the other corresponding to \( - \) sign. We have numerically analyzed the dispersion relation (7) for both \(+\) and \(-\) signs, and have shown how the dispersion properties of these two modes are modified by the parameter \( \mu \) which is not the same for all dusty plasma systems, but varies from \( \sim 10^{-4} \) to \( \sim 10^{-2} \) \([\text{Chow et al., 1993; Mendis and Rosenberg, 1994; Mendis and Horányi, 1991; Horányi, 1996}] \). The results are displayed in Figure 1. The lower (upper) plot shows how the dispersion properties of the mode corresponding to \( + \) (\(-\)) sign in (7) are modified by changing the parameter \( \mu \). To interpret these two modes more precisely, we assume that \( 12\mu k^2\lambda_{D1}^2 \ll (1 + \mu + 3k^2\lambda_{D1}^2)^2 \) which is valid for either \( \mu \ll 1 \) or \( k\lambda_{D1} \ll 1 \). Thus, the modes corresponding to \( + \) and \(-\) signs of (7) can be approximated as \( \omega_+ \approx \omega_{p2}\sqrt{1 + \mu + 3k^2\lambda_{D1}^2} \) and \( \omega_- \approx \sqrt{3k\lambda_{D1}} \sqrt{1 + \mu + 3k^2\lambda_{D1}^2} \). These mean that the modes corresponding to \( + \) (\(-\)) sign in (7) represent, respectively, the dust-Langmuir waves (associated with the oscillation and thermal pressure of light positive particles) and the dust-acoustic waves (in which the massive negative particles provide the inertia and the pressure of light positive particles provides the restoring force). The interpretation presented here is analogous to an electron ion plasma, i.e. here positive (negative) dust particles play the role of electrons (ions). This interpretation is only valid when either \( \mu \ll 1 \) or \( k\lambda_{D1} \ll 1 \). However, unlike electron-ion plasma \( \mu \ll 1 \) may not always be valid and \( \mu \) is not constant but varies depending on the dusty plasma system we consider. This is one of the important features of our present model which cannot be visualized in an electron-ion plasma.

4. Nonlinear Waves
[10] To investigate the nonlinear properties of the electrostatic waves described in section 3, we must consider the nonlinear terms in (1)–(6). Therefore, in order to study the fully nonlinear (arbitrary amplitude solitary) waves, we
employ here the pseudo-potential approach [Bernstein et al., 1957; Sagdeev, 1966] by assuming that all dependent variables depend on a single variable $x = x/C_0 M t$, where $M$ is the Mach number (solitary wave speed/$C_s$). Using this transformation ($x = x/C_0 M t$) along with the steady state condition ($\partial/\partial t = 0$) and appropriate boundary conditions for localized perturbations (viz. $n_j \to 1$, $p \to 1$, $u_j \to 0$, $\Phi \to 0$, and $d\Phi/d\xi \to 0$ at $\xi \to \pm \infty$), we can reduce (1)-(6) to [Bernstein et al., 1957; Sagdeev, 1966; Mamun, 1999; Shukla and Mamun, 2002]:

\[
\frac{1}{2} \left( \frac{d\Phi}{d\xi} \right)^2 + V(\Phi, M, \mu) = 0,
\]

where

\[
V(\Phi, M, \mu) = -M \sqrt{2} \left[ 3 + \mu M^2 \right]^{1/2} + 2\Phi + V_1 + V_2 + C,
\]

in which

\[
V_1 = -\frac{M \sqrt{2}}{\sqrt{2}} \left[ 3 + \mu M^2 \right]^{1/2} - 2\Phi \pm \sqrt{(3 + \mu M^2 - 2\Phi)^2 - 12\mu M^2},
\]

\[
V_2 = -2\sqrt{2} M^3 [3 + \mu M^2]^{1/2} - 2\Phi \pm \sqrt{(3 + \mu M^2 - 2\Phi)^2 - 12\mu M^2}^{1/2},
\]

and $C$ is an integration constant which we choose in such a way that $V(\Phi, M, \mu) = 0$ at $\Phi = 0$. We note that $+ (-)$ corresponds to the dust-Langmuir (dust-acoustic) waves. The nonlinear equation (8) can be regarded as an “energy integral” of an oscillating particle of unit mass, with velocity $d\Phi/d\xi$, and position $\Phi$ [Bernstein et al., 1957; Sagdeev, 1966]. This equation is valid for arbitrary amplitude dust-acoustic or dust-Langmuir waves in the steady state. However, to study the nonlinear propagation of small amplitude dust-acoustic (dust-Langmuir) waves one...

Figure 2. The behaviour of $V(\Phi, M, \mu)$ for $\mu = 0.01$ and different values of $M$. The lower plot, where $M = 1.725$ (solid curve), $M = 1.735$ (dotted curve), and $M = 1.740$ (dashed curve), shows the critical Mach number for which a potential well in the negative $\Phi$-axis (corresponding to a solitary wave with a negative potential) develops. The upper plot, where $M = 2.88$ (solid curve), $M = 2.90$ (dotted curve), and $M = 2.92$ (dashed curve), shows the upper limit of the Mach number for which a potential well in the negative $\Phi$-axis develops.

Figure 3. The behaviour of $V(\Phi, M, \mu)$ for $\mu = 0.1$ and different values of $M$. The lower plot, where $M = 1.655$ (solid curve), $M = 1.665$ (dotted curve), and $M = 1.670$ (dashed curve), shows the critical Mach number for which a potential well in the negative $\Phi$-axis develops. The upper plot, where $M = 1.96$ (solid curve), $M = 1.98$ (dotted curve), and $M = 1.99$ (dashed curve), shows the upper limit of the Mach number for which a potential well in the negative $\Phi$-axis develops.
can derive the Korteweg-de Vries (Schrödinger) equation [Dodd et al., 1982] by using the reductive perturbation method on (1)–(6).

[11] It is obvious from (9) that $V(\Phi, M, \mu) = 0$ at $\Phi = 0$. Thus, the solitary wave solutions of (8) exist if (i) $(dV/d\Phi)_{\Phi=0} = 0$, (ii) $(d^2V/d\Phi^2)_{\Phi=0} < 0$ so that the fixed point at the origin is unstable, i.e., $V(\Phi) < 0$ for $0 < |\Phi| < |\Phi_m|$, and (iii) $V(\Phi_m) = 0$, where $\Phi_m$ is the amplitude of the solitary waves. Because of the complicated form of the pseudo-potential $V$ (a complicated function of $M$ and $\mu$), it is not possible to obtain a simple analytical condition for the existence of solitary wave solutions of (8). However, by numerically solving (8) or simply analyzing the pseudo-potential $V(\Phi, M, \mu)$ one can find the parametric regimes for which the acoustic-like solitary wave [Stenflo et al., 1989] solutions of (8) exist. We have numerically analyzed the pseudo-potential $V(\Phi, M, \mu)$ given by (9) for different possible values of $M$ and $\mu$, and found that the solitary wave solutions of (8) exist for the minus sign of (9) (which corresponds dust-acoustic waves), but not for $+$ of (9) (which corresponds to dust-Langmuir waves). We have shown that when $\mu < 0.01$ the dust-acoustic solitary waves exist for $1.72 < M < 2.92$ (cf. Figure 2) and when $\mu = 0.1$ the dust-acoustic solitary waves exist for $1.67 < M < 1.98$ (cf. Figure 3). Figures 2 and 3 also show that as we increase $\mu$, both the minimum and maximum values of the Mach number for which dust-acoustic solitary waves exist decrease, particularly, the upper limit of the Mach number decreases more significantly. We have also found that the amplitude of the solitary waves increases with increasing the value of the parameter $\mu$.

5. Discussion

[12] We have presented a new dusty plasma model that is appropriate for cometary tails [Mendis and Horányi, 1991; Chow et al., 1993; Mendis and Rosenberg, 1994; Horányi, 1996]. Our present dusty plasma model is unique in that we have considered charged dust grains of opposite polarity, without electrons and ions in the ambient plasma. It is found that such a two-component dusty plasma supports dust-Langmuir and dust-acoustic dispersive waves.

[13] When we have considered a highly nonlinear regime, we have found that dust-acoustic waves propagate in the form of solitary structures that are associated with a negative potential. We have shown that when $\mu = 0.01$ the dust-acoustic solitary waves exist for $1.72 < M < 2.92$ (cf. Figure 2) and when $\mu = 0.1$ these exist for $1.67 < M < 1.98$ (cf. Figure 3). It is also found that as we increase $\mu$, both the minimum and maximum values of the Mach number for which dust-acoustic solitary waves exist decrease, particularly, the upper limit of the Mach number decreases more significantly. Besides, the amplitude of the solitary waves increases with increasing the value of the parameter $\mu$.

[14] At present there do not exist any conclusive evidence of localized electrostatic potential structures in cometary tails. However, it ought to be stressed that solitary negative potentials may trap positively charged dust particles, which, in turn, attract dust particles of opposite polarity to form larger sized dust particles or to be coagulated into extremely large sized neutral dust in cometary tails. Thus, the results of the present investigation should help to identify the origin of charge separation as well as dust coagulation in a plasma containing positive and negative dust particles.

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References


