Dust–Acoustic Shocks in a Strongly Coupled Dusty Plasma

Padma K. Shukla and A. A. Mamun

***Abstract*—It is shown that the nonlinear propagation of the dust-acoustic waves in a strongly coupled dusty plasma is governed by a Korteweg–de Vries–Burgers (K–dV–Burgers) equation. The latter is derived from a set of generalized hydrodynamic equations for strongly correlated dust grains, as well as the Boltzmann distribution for electrons and ions. Possible stationary solutions of the K–dV–Burgers equation are represented in terms of monotonic/oscillatory shock profiles.**

***Index Terms*—Dust–acoustic shocks, Korteweg–de Vries–Burgers equation, strongly coupled dusty plasma.**

ABOUT A DECADE ago, Rao *et al.* [1] theoretically predicted the existence of linear and nonlinear dust–acoustic waves (DAWs) in an unmagnetized dusty plasma, in which the dust grains are weakly correlated so that the electrostatic interaction energy of shielded grains is much smaller than the kinetic energy of the dust particles. In the DAWs, the restoring force comes from the pressures of the inertialess electrons and ions, while the inertia is provided by the dust particle mass. Since the frequency of the DAWs is much smaller than the dust plasma frequency, they appear on a very long timescale so that their video images are possible [2], [3]. It has been shown by several authors [4], [5] that the DAWs suffer collisionless, non-Landau damping when one accounts for the dust charge fluctuation dynamics. The latter also causes the damping of the dust–acoustic soliton [6] as well as it is also responsible for the dust–acoustic shocks [7]. At the same time, it has also been shown by Shukla and Silin [8] that an unmagnetized weakly coupled dusty plasma supports the dust–ion–acoustic wave (DIAW) whose phase velocity is much smaller (larger) than the electron (ion) thermal speed. The frequency of the DIAWs is much larger (smaller) than the dust (ion) plasma frequency. The DIAWs are not subjected to the ion Landau damping, but they do suffer a collisional damping [9] due to the dust–ion interaction. The latter produces a kinematic viscosity which is responsible for the formation of dust ion–acoustic (DIA) shocks and holes [9]. The detailed theoretical models for the DIA shocks and DIA holes are presented by Shukla [10]. The formation of ion acoustic shocks in a dusty plasma has been experimentally observed by Luo *et al.* [11], [36]. Popel *et al.* [12] have discussed the formation of DIA shocks, taking into account the dust charge fluctuation dynamics. Thus, the properties of one-dimensional linear and nonlinear waves in a weakly coupled unmagnetized dusty plasma are well understood.

However, in a strongly coupled dusty plasma [13], [37], the intergrain spacing is of the order of the effective dusty plasma Debye radius, and the electrostatic interaction energy of the shielded grains is much larger than the kinetic energy of the dust grains. Here, one has the possibility of dust lattice waves (DLWs) [14]–[17] in which the restoring force comes from the shielded Coulomb force between the nearest neighboring dust grains, while the inertia is provided by the dust particle mass. It has been found that the dynamics of the nonmodulated (modulated) DLWs is governed by the Korteweg–de Vries (K–dV) [14], [15] (the nonlinear Shrödinger (NLS) [18]) equation. It has been shown that both the K–dV and NLS equations admit soliton solutions. The DLWs are experimentally observed by Homann *et al.* [19].

Recently, a number of authors have considered the linear properties of longitudinal DAWs in a strongly coupled unmagnetized dusty plasma within the framework of either a generalized hydrodynamic (GH) model [20], [21] or the quasilocalized charge (QLC) approximation [22], the local field correction (LFC) method [23], [38]. The transverse DAWs have also received a great deal of interest in understanding them theoretically [24]–[26], and experimentally [27]. Ohata and Hamguchi [24] have investigated the transverse DAWs in a Yukawa dusty plasma system by molecular dynamics simulation. It turns out that the findings of Ohata and Hamaguchi [24] are in good agreement with the GH model [20].

In this paper, we study the nonlinear propagation of the DAWs in a strongly coupled unmagnetized dusty plasma. We employ the reductive perturbation technique on the GH equations [20], [21] and derive a K–dV–Burgers equation, which admits monotonic/oscillatory shock solutions.

We consider the nonlinear propagation of the DAWs in an unmagnetized strongly coupled dusty plasma whose constituents are electrons, ions, and negatively charged, massive dust grains. Thus, at equilibrium we have $n_{i0} = Z_d n_{d0} + n_{e0}$, where n_{i0}, n_{d0}, and n_{e0} are the unperturbed ion, dust, and electron number densities, respectively, and Z_d is the number of charges residing on the dust grain surface.

We assume that the electrons and the ions are weakly coupled due to their higher temperatures and smaller electric charges. Thus, in the presence of low phase velocity (in comparison with the electron and ion thermal velocities) DAWs, the electron and ion number densities obey a Boltzmann distribution. On the other hand, we assume that the dust grains are strongly coupled because of their lower temperature and larger electric charge.

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The dynamics of the nonlinear DAWs in our strongly coupled dusty plasma is governed by the well-known GH equations [20], [21], which are

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0 \quad (1)$$

$$\left(1 + \tau_m \frac{\partial}{\partial t}\right)\left[n_d\left(D_t u_d + \nu_{dn} u_d - \frac{\partial \phi}{\partial x}\right)\right] = \eta_l \frac{\partial^2 u_d}{\partial x^2} \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_d + \alpha_e e^{\phi} - \alpha_i e^{-\sigma\phi} \quad (3)$$

where

- n_d dust particle number density normalized by its equilibrium value n_{d0};
- u_d dust fluid velocity normalized by the dust–acoustic speed $C_d = (Z_d T_e/m_d)^{1/2}$ (T_e is the electron temperature in units of the Boltzmann constant and m_d is the dust mass);
- ϕ electrostatic wave potential normalized by T_e/e (e is the magnitude of the electron charge);
- α_e $= n_{e0}/Z_d n_{d0}$;
- α_i $= n_{i0}/Z_d n_{d0}$;
- σ ($=T_i/T_e$) ratio between the ion and electron temperatures.

The time and space variables are in the units of the dust plasma period $\tau_d = (m_d/4\pi n_{d0} Z_d^2 e^2)^{1/2}$ and the Debye length $\lambda_{Dd} = (T_e/4\pi Z_d n_{d0} e^2)^{1/2}$, respectively, and $D_t = \partial/\partial t + u_d \partial/\partial x$. Furthermore, ν_{dn} is the dust-neutral collision frequency normalized by the dust plasma frequency τ_d^{-1}, τ_m is the viscoelastic relaxation time normalized by the dust plasma period τ_d, $\eta_l = (\tau_d/m_d n_{d0}\lambda_{Dd}^2)[\eta_t + (4/3)\zeta_t]$, where η_t and ζ_t are transport coefficients of shear and bulk viscosities, is the normalized longitudinal viscosity coefficient. There are various approaches [28]–[33] for calculating these transport coefficients. The viscoelastic relaxation time τ_m is given by [28], [30]

$$\tau_m = \eta_l \frac{T_e}{T_d}\left[1 - \mu_d + \frac{4}{15}u(\Gamma)\right]^{-1} \quad (4)$$

where

$$\mu_d - \frac{1}{k_B T_d}\frac{\partial P_d}{\partial n_d} = 1 + \frac{1}{3}u(\Gamma) + \frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial \Gamma} \quad (5)$$

is the compressibility [28], [29] and $u(\Gamma)$ is a measure of the excess internal energy of the system and is calculated for weakly coupled plasmas ($\Gamma < 1$) as [32] $u(\Gamma) \simeq -(\sqrt{3}/2)\Gamma^{3/2}$. To express $u(\Gamma)$ interms of Γ for a range of $1 < \Gamma < 100$, Slattery *et al.* [33] have analytically derived a relation

$$u(\Gamma) \simeq -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81 \quad (6)$$

where a small correction term due to finite number of particles is neglected. The dependence of the other transport coefficient η_l on Γ is somewhat more complex and cannot be expressed in such a closed analytical form. However, tabulated/graphical results of their functional behavior derived from molecular dynamic (MD) simulations and a variety of statistical schemes are available in the literature [28], [29]. Typical values [28] of η_l are $1.04a_d^2/\lambda_{Dd}^2$ for $\Gamma = 1$, $0.08a_d^2/\lambda_{Dd}^2$ for $\Gamma = 10$, and $0.3a_d^2/\lambda_{Dd}^2$ for $\Gamma = 160$.

In order to derive a dynamical equation for the DA shock waves from our basic equations (1)–(3), we employ the reductive perturbation technique. We introduce the stretched coordinates

$$\xi = \epsilon^{1/2}(x - u_0 t) \quad (7)$$

$$\tau = \epsilon^{3/2} t \quad (8)$$

where ϵ is a smallness parameter measuring the weakness of the amplitude or dispersion and u_0 is the shock wave phase velocity (normalized to C_d), and expand the variables n_d, u_d, and ϕ about the unperturbed states in power series of ϵ, *viz.*

$$n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \cdots \quad (9)$$

$$u_d = \epsilon u_d^{(1)} + \epsilon^2 u_d^{(2)} + \cdots \quad (10)$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots. \quad (11)$$

Substituting (7)–(11) into (1)–(3), we obtain from the equations of the lowest order in ϵ, $u_d^{(1)} = -\phi^{(1)}/u_0$, $n_d^{(1)} = -\phi^{(1)}/u_0^2$, and $u_0 = (\alpha_e + \sigma\alpha_i)^{-1/2}$. To next order in ϵ, we have

$$\frac{\partial n_d^{(1)}}{\partial \tau} - u_0 \frac{\partial n_d^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi}\left[n_d^{(1)} u_d^{(1)}\right] + \frac{\partial u_d^{(2)}}{\partial \xi} = 0 \quad (12)$$

$$(1 + \nu_{dn}\tau_m)\frac{\partial u_d^{(1)}}{\partial \tau} - u_0 \frac{\partial u_d^{(2)}}{\partial \xi} - \frac{\partial \phi^{(2)}}{\partial \xi} + (1 - \nu_{dn}\tau_m) u_d^{(1)} \frac{\partial u_d^{(1)}}{\partial \xi} - \eta_0 \frac{\partial^2 u_d^{(1)}}{\partial \xi^2} = 0 \quad (13)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} - \frac{1}{u_0^2}\phi^{(2)} - n_d^{(2)} = \frac{1}{2}(\alpha_e - \alpha_i \sigma^2)\left[\phi^{(1)}\right]^2 \quad (14)$$

where we have assumed that $\eta_l \sim \epsilon^{1/2}\eta_0$ [10]. It may be noted that because of the ordering we have used, in (13) the terms containing $\nu_{dn}\tau_m$ survive and the terms that depend only on τ_m are factored out. By eliminating $n_d^{(2)}$, $u_d^{(2)}$, and $\phi^{(2)}$ from (12)–(14) we readily obtain the K–dV–Burgers equation

$$A^{-1}\frac{\partial \phi^{(1)}}{\partial \tau} + \phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + \beta \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = \mu \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} \quad (15)$$

where

- A $= (u_0^3\delta/2)(1 + \nu_{dn}\tau_m/2)^{-1}$;
- μ $= \eta_0/\delta u_0^3$;
- β $= 1/\delta$;
- δ $= [(\nu_{dn}\tau_m - 3)/u_0^4 - \alpha_e + \alpha_i\sigma^2]$.

As $\nu_{dn} > 0$, $\tau_m > 0$, $\eta_0 > 0$, and $u_0 = (\alpha_e + \sigma\alpha_i)^{-1/2} > 0$, the sign of the coefficients A, β, and μ are determined by the sign of δ, which can be expressed as $\delta = (\nu_{dn}\tau_m - 3)(\alpha_e + \sigma\alpha_i)^2 - \alpha_e + \alpha_i\sigma^2 = \alpha_e[(\nu_{dn}\tau_m - 3)\alpha_e - 1] + \alpha_i^2\sigma^2(\nu_{dn}\tau_m - 3 + Z_d n_{d0}/n_{i0}) + 2(\nu_{dn}\tau_m - 3)\alpha_e\alpha_i\sigma$. It is obvious that for a strongly coupled dusty plasma with a significant background of neutrals, we have $\nu_{dn}\tau_m \gg 1$, i.e., $\delta > 0$, which corresponds to $A > 0$, $\mu > 0$, and $\beta > 0$, whereas for a weakly coupled or a collisionless dusty plasma ($\nu_{dn}\tau_m \to 0$), we have $\delta < 0$, which corresponds to $A < 0$, $\mu < 0$, and $\beta < 0$.

Let us now discuss possible stationary solutions of the K–dV–Burgers equation. For our purposes, we first transform the independent variables ξ and τ to $\zeta = \xi - U_0\tau$ and $\tau = \tau$, where U_0 is a constant velocity normalized by C_d, and find a third order ordinary differential equation for $\phi^{(1)}(\zeta)$. The latter can be integrated once, obtaining

$$\beta\frac{\partial^2\phi^{(1)}}{\partial\zeta^2} - \mu\frac{\partial\phi^{(1)}}{\partial\zeta} + \frac{1}{2}[\phi^{(1)}]^2 - \frac{U_0}{A}\phi^{(1)} = 0 \qquad (16)$$

where we have imposed the appropriate boundary conditions, viz. $\phi^{(1)} \to 0$, $d\phi^{(1)}/d\zeta \to 0$, $d^2\phi^{(1)}/d\zeta^2 \to 0$ at $\zeta \to \infty$. We can now easily [34] show that (16) describes a shock wave whose velocity (in the reference frame under consideration) U_0 is related to the extreme values $\phi^{(1)}(-\infty) - \phi^{(1)}(\infty) = \varphi$ by $U_0/A = \varphi/2$. Thus, in the rest frame the velocity (normalized to C_d) of the shock wave is $(1 + A\varphi/2)$. The nature of the structure of the shock wave depends on the relation between the dispersive and dissipative parameters β and μ.

We first consider a situation where the dissipative term is dominated over the dispersive term, i.e., where we can neglect the dispersive term. In this case we can express (16) as

$$\left[\phi^{(1)} - \frac{U_0}{A}\right]\frac{\partial\phi^{(1)}}{\partial\zeta} = \mu\frac{\partial^2\phi^{(1)}}{\partial\zeta^2}. \qquad (17)$$

Equation (17) can be easily integrated, using the condition that $\phi^{(1)}$ is bounded as $\zeta \to \pm\infty$, to obtain [35]

$$\phi^{(1)} = \frac{U_0}{A}\left[1 - \tanh\left(\frac{U_0}{2A\mu}(\xi - U_0\tau)\right)\right]. \qquad (18)$$

Equation (18) represents a monotonic shock solution with the shock speed, the shock height, and the shock thickness given by U_0, $\phi_0 = U_0/A$, and $\Delta = A\mu/U_0$, respectively. The shock solution appears because of the dissipative term which is proportional to the longitudinal viscosity coefficient (η_l). The shock height $\phi_0 = U_0/A$ and the shock thickness $\Delta = A\mu/U_0$ are, respectively,

$$\phi_0 = \frac{2U_0u_0(1 + \frac{1}{2}\nu_{dn}\tau_m)}{\nu_{dn}\tau_m + (\alpha_i\sigma^2 - \alpha_e)u_0^4 - 3} \qquad (19)$$

and

$$\Delta = \frac{\eta_0}{2(1 + \frac{1}{2}\nu_{dn}\tau_m)}. \qquad (20)$$

Equations (19) and (20) indicate that the shock height (ϕ_0) is independent of longitudinal viscosity coefficient (η_0), but the shock thickness (Δ) is directly proportional to the latter (i.e., the shock thickness increases with increasing the longitudinal viscosity coefficient). Equation (20) also means that for a strongly coupled and highly collisional dusty plasma regime, where $\nu_{dn}\tau_m \gg 1$, the shock thickness is inversely proportional to τ_m or ν_{dn} (i.e., the shock thickness decreases with increasing the viscoelastic relaxation time or the dust-neutral collision frequency). To see how the shock height explicitly depends on τ_m and ν_{dn}, we can use an appropriate assumption $T_e \simeq T_i$ and can express (19) as

$$\phi_0 = \frac{2U_0\left(\dfrac{Z_dn_{d0}}{n_{i0}+n_{e0}}\right)^{1/2}\left(1 + \dfrac{1}{2}\nu_{dn}\tau_m\right)}{\nu_{dn}\tau_m - 3 + \left(\dfrac{Z_dn_{d0}}{n_{i0}+n_{e0}}\right)^2}. \qquad (21)$$

As $Z_dn_{d0}/(n_{i0} + n_{e0}) < 1$, (21) implies that $\phi_0 < 0$ for a weakly collisional dusty plasma regime ($\nu_{dn}\tau_m \ll 1$), whereas $\phi_0 > 0$ for a highly collisional dusty plasma regime ($\nu_{dn}\tau_m \gg 1$). It is also seen from (21) that in both the situations the shock height is independent of the viscoelastic relaxation time (τ_m) or the dust-neutral collision frequency (ν_{dn}).

We now discuss the effects of the dispersive term on the shock solution of (16). When μ is extremely small, the shock wave will have an oscillatory profile in which the first few oscillations at the wave front will be close to solitons [34] moving with velocity U_0. If μ is increased and it is larger than a certain critical value μ_c, the shock wave will have a monotonic behavior. To determine the values of the dissipation coefficient μ corresponding to monotonic or oscillating shock profiles, we investigate the asymptotic behavior of the solutions of (16) for $\zeta \to -\infty$. We first substitute $\phi^{(1)}(\zeta) = \varphi + \psi(\zeta)$, where $\psi \ll \varphi$, into (16), and then linearize it with respect to ψ in order to obtain

$$\beta\frac{\partial^2\psi}{\partial\zeta^2} - \mu\frac{\partial\psi}{\partial\zeta} + \frac{U_0}{A}\psi = 0. \qquad (22)$$

The solutions of (22) are proportional to e^{px}, where p is given by

$$p = \frac{\mu}{2\beta} \pm \left(\frac{\mu^2}{4\beta^2} - \frac{U_0}{A\beta}\right)^{1/2}. \qquad (23)$$

It turns out that the shock wave has a monotonic profile for $\mu > \mu_c$ and an oscillating profile for $\mu < \mu_c$, where $\mu_c = (4\beta U_0/A)^{1/2} = (2/\delta u_0)(2U_0/u_0)^{1/2}(1 + \nu_{dn}\tau_m/2)^{1/2}$. It is important to mention here that μ_c is independent of the longitudinal viscosity coefficient (η_l). However, for $\nu_{dn}\tau_m \gg 1$, μ_c increases as the viscoelastic relaxation time (τ_m) or the dust-neutral collision frequency (ν_{dn}) increases. For $\mu \ll \mu_c$, the stationary solution of (15) for $\phi^{(1)}$ is given by [34]

$$\phi^{(1)} = \varphi + C\,\exp\left(\frac{Z\mu}{2\beta}\right)\cos\left(Z\sqrt{\frac{U_0}{A\beta}}\right) \qquad (24)$$

where $Z = \xi - U_0\tau$ and C is a constant. It should be noted here that for a weakly collisional dusty plasma ($\nu_{dn}\tau_m \to 0$), we have $\delta < 0$, which corresponds to $A < 0$, $\mu < 0$, and $\beta < 0$. In this case, one can easily show that the solution of (15) is exactly similar to (24), with the opposite sign of the constant C and φ.

To summarize, we have investigated the properties of DA shocks in a strongly coupled unmagnetized dusty plasma. We have employed the reductive perturbation technique on the GH equations and have derived a K–dV–Burgers equation, which admits shock solutions. It turns out that due to strong correlations between the dust grains, there may exist dust–acoustic shocks which have either oscillatory or monotonic behavior. It should be noted that the model we have considered is not valid for other (non-Yukawa) interactions, which may arise from dipoles, wakes, etc. It may be added that the shock structures we found in our present analysis are different from those observed by recent laboratory experiment and analytical analysis of Nakamura *et al.*[9] who considered weakly coupled dusty plasma and neglected the dynamics of dust particles (i.e., considered the static dust grains in their dusty plasma model).

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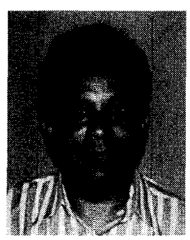

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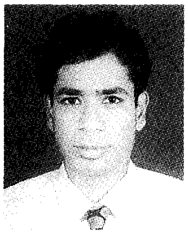
He moved from India to Umeå in January 1972 and later (in January 1973) joined the Faculty of Physics and Astronomy, Ruhr University Bochum, Germany. He was also conferred a professorship at the Department of Plasma Physics, Umeå University in December 1997. His main research interests include theoretical plasma physics, nonlinear structures in plasmas and fluids, and convective processes in space physics and astrophysics. His present interests involve the physics of dusty plasmas, astroparticle physics (collective neutrino plasma interaction), and nonlinear processes in planetary magnetospheres and astrophysics. He is an Associate Editor with IEEE TRANSACTIONS OF PLASMA SCIENCE and the *Journal of Plasma Physics*. At present, he is also serving as an Editorial Board Member of *Plasma Physics and Controlled Fusion* and the *Journal of Research in Physics*. He served as an Editorial Board Member of *Planetary and Space Science* during the period 1993–1998. He has organized many international conferences in plasma physics at the Abdus Salam International Center for Theoretical Physics, Trieste, Italy, as well as at The University of Algarve, Faro,Portugal. He also serves as advisory board member for a number of international conferences/workshops, and is a member of several international scientific organizations, including APS and Max–Planck Institut für extraterrestrische Physik,Garching, Germany. In addition, he has been a Co-Investigator/PI and a Coordinator for many projects from the European Union, NATO, and DLR/ISRO.



A. A. Mamun was born in 1966 in Dhaka, Bangladesh. He received the B.Sc. and the M.Sc. degrees from Jahangirnagar University, Dhaka, Bangladesh in 1986 and 1987, respectively. He was awarded the Commonwealth Scholarship by its Commission in London to study for the Ph.D. degree. He received the Ph.D. degree in plasma physics from the University of St. Andrews, Fife, Scotland, in 1996. He has recently been awarded the Alexander von Humboldt post-doctoral Research Fellowship to carry out the advanced research in dusty plasma physics in Ruhr Universität Bochum, Germany.

He is currently an Associate Professor of Physics at Jahangirnagar University, and is the Regular Associate of the International Centre for Theoretical Physics, Trieste, Italy. His main research field is theoretical and computational plasma physics which involves collective processes in dusty plasmas, electron-positron plasma, laser-produced plasma, semiconductor plasma, etc. He has authored more than 50 papers published in the best scientific journals on linear and nonlinear waves in dusty plasmas.

Dr. Mamun has been awarded the Best Young Scientist of Bangladesh for the year 1992 by the Bangladesh Association for the Advancement of Science for the best research presentation. He has also been awarded the Best Young Scientist Prize for the year of 2000 by the International Center for Theoretical Physics, Trieste, Italy, in recognition of his pioneering contribution to the study of nonlinear phenomena in complex (dusty) plasmas. He is a member of the American Association for Advancement of Science by the special invitation.