Spherical and cylindrical dust acoustic solitary waves

A.A. Mamun * ,1, P.K. Shukla

Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr Universität Bochum, D-44780 Bochum, Germany

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Abstract

The nonlinear propagation of cylindrical and spherical dust acoustic (DA) waves in an unmagnetized dusty plasma is considered. By employing the reductive perturbation technique, a modified Korteweg–de Vries (K–dV) equation is derived from the dust continuity and momentum equations, Poisson’s equation as well as Boltzmann electrons and ions. Numerical solutions of the modified K–dV equation are obtained. It has been found that the propagation characteristics of the cylindrical and spherical DA solitary waves significantly differ from those of one-dimensional DA solitary waves.

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About a decade ago, Rao et al. [1] theoretically predicted dust acoustic (DA) waves in which the inertia is provided by the dust particle mass and the restoring force comes from the pressures of inertial electrons and ions. The theoretical prediction of Rao et al. [1] has been conclusively verified by a number of laboratory experiments [2,3]. The linear properties of the DA waves are now well understood from both theoretical and experimental points of view [1–4]. Recently, nonlinear waves associated with the DA waves, particularly the DA solitary waves [1], have received a great deal of attention for understanding the basic properties of localized electrostatic perturbations in space and laboratory dusty plasmas [5–9]. The DA solitary waves have been investigated by a number of authors by assuming different unmagnetized dusty plasma models [1,10–12]. However, all these investigations are limited to one-dimensional (planar) geometry which may not be a realistic situation in space and laboratory devices, since the waves observed in space (laboratory devices) are certainly not infinite (bounded) in one dimension.

In this Letter, we present an investigation of cylindrical and spherical DA solitary waves in an unmagnetized dusty plasma. We show here how the DA solitary waves in cylindrical and spherical geometries differ qualitatively from that in one-dimensional planar geometry.

We focus on cylindrical and spherical DA solitary waves in an unmagnetized dusty plasma whose constituents are negatively charged cold dust fluid and Boltzmann electrons and ions. The nonlinear dynamics of low phase speed (in comparison with the electron and ion thermal speeds) DA waves in cylindrical and spherical geometries is governed by

\[
\frac{\partial n_d}{\partial t} + \frac{1}{r^\nu} \frac{\partial}{\partial r} \left( r^\nu n_d u_d \right) = 0,
\]

(1)

* Corresponding author.

E-mail address: am@tp4.ruhr-uni-bochum.de (A.A. Mamun).

1 Permanent address: Department of Physics, Jahangirnagar University, Savar, Dhaka, Bangladesh.

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\[
\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial r} = \frac{\partial \varphi}{\partial r} \tag{2}
\]

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \varphi}{\partial r} \right) = n_d + \mu_e \exp(\sigma_i \varphi) - \mu_i \exp(-\varphi), \tag{3}
\]

where \( v = 1, 2 \) for cylindrical and spherical geometries, respectively, \( n_d \) is the dust particle number density normalized by its equilibrium value \( n_{e0} \), \( u_d \) is the dust fluid velocity normalized by \( C_d = (Z_d e/k_BT_1/m_d)^{1/2} \), and \( \varphi \) is the electrostatic wave potential normalized by \( k_BT_1/e \). The time and space variables are in units of the dust plasma period \( \omega_{pd} = (m_d/4\pi n_{e0} Z_d^2 e^2)^{1/2} \) and the Debye length \( \lambda_D = (k_BT_1/4\pi Z_d n_{e0} e^2)^{1/2} \), respectively. We have denoted \( \mu_e = n_{e0}/Z_d n_{e0} = 1/(\delta - 1) \), \( \mu_i = n_{i0}/Z_d n_{e0} = \delta/(\delta - 1) \), \( \delta = n_{e0}/n_{i0} \), and \( \sigma_i = T_i/T_e \).

To investigate ingoing solutions of Eqs. (1)–(3), we introduce the stretched coordinates \[13\] and develop equations in various powers of \( \epsilon \).

\[
n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \cdots, \tag{4a}
\]

\[
u = \epsilon u^{(1)} + \epsilon^2 u^{(2)} + \cdots, \tag{4b}
\]

\[
\varphi = \epsilon \varphi^{(1)} + \epsilon^2 \varphi^{(2)} + \cdots. \tag{4c}
\]

and form the stationary solution of Eq. (6) without the \( \epsilon \) term.

If we compare Eq. (6) with Eq. (13) of Ref. \[12\], it is obvious that the term \( (\nu/2\tau)\varphi^{(1)} \) in Eq. (6) is due to the effect of the cylindrical or spherical geometry.

We numerically solve Eq. (6) by using a two-level finite difference approximation method \[13\] and study the effects of spherical and cylindrical geometries on time-dependent DA solitary waves. The results showing spherical and cylindrical solitary waves are shown in the adjoining figure. The initial condition that we have used in our numerical analysis is in the form of the stationary solution of Eq. (6) without the term \( (\nu/2\tau)\varphi \), i.e.,

\[
\varphi^{(1)} = -\varphi_m^{(1)} \text{sech}^2(\zeta/\Delta), \tag{8}
\]

where \( \varphi_m^{(1)} = 3/A \) and \( \Delta = \sqrt{4B} \). Numerical solutions of Eq. (6) (shown in Fig. 1) reveals that for a large value of \( \tau \) the term \( (\nu/2\tau)\varphi^{(1)} \) which is due to the effect of the cylindrical or spherical geometry, is no longer dominant. However, as the value of \( \tau \) decreases, the term \( (\nu/2\tau)\varphi^{(1)} \) becomes dominant and both spherical and cylindrical solitary waves differ from one-dimensional solitary waves. It is also found that as the value of \( \tau \) decreases, the amplitude of these localized pulses increases. Furthermore, the cylindrical solitary waves travel faster than the one-dimensional solitary waves but slower than the spherical ones, and that the amplitude of cylindrical solitary waves is larger than that of the one-dimensional solitary waves but slightly smaller than that of the spherical ones. In conclusion, we mention that the present results may help to understand the salient features of multi-dimensional DA solitary waves when data for space and laboratory observations become available.
Fig. 1. Time evolution of spherical and cylindrical solitary waves: $\psi^{(1)}$ versus spatial coordinate $\zeta$ at times $\tau = -31.6$, $\tau = -16.6$, and $\tau = -6.6$ for $\delta = 10$ and $\sigma_i = 0.1$. The solid line corresponds to spherical ($\nu = 2$) solitary waves and the dashed line corresponds to cylindrical ($\nu = 1$) solitary waves.

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