Plasma voids (holes) in a dusty plasma

A.A. Mamun a,1, P.K. Shukla a,*, R. Bingham b

a Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr Universität Bochum, D-44780 Bochum, Germany
b Rutherford Appleton Laboratory, Chilton, Didcot, Oxfordshire, OX 11 0QX, England, UK

Received 1 April 2002; received in revised form 4 April 2002; accepted 8 April 2002

Communicated by V.M. Agranovich

Abstract

A theory for plasma voids (large amplitude plasma holes) in a dusty plasma is presented. Specifically, it is proposed that the plasma voids are self-consistent solutions of Poisson’s equation in which the electron density response is Boltzmannian and the ion density response is non-Maxwellian due to the ion trapping in large amplitude plasma potentials. Poisson’s equation, with appropriate electron and ion number densities, is then reduced to an energy integral which provides criteria for the existence of the plasma voids. The latter are characterized as regions of significant ion and electron density depletions in association with negative plasma potentials. The voids shrink when the dust grains are added into the plasma, and they tend to disappear if the dust number density is sufficiently high. © 2002 Elsevier Science B.V. All rights reserved.

PACS: 52.35.Fp; 52.25.Zb; 52.35.Mw; 52.35.Sb

The physics of charged dust particles, which are ubiquitous in most space and laboratory plasmas [1–5], has become an outstanding and challenging research topic not only because it has introduced a great variety of new phenomena associated with waves and instabilities [3–13], but also because it provided a number of new experimental events/facts, viz. the formation of plasma crystals [14–16], Mach cones [17], voids [18–20], etc. Though the entire plasma volume is usually filled by the dust particles, dust free regions (known as dust voids) inside the dust cloud have been observed in silane [18] and other dust laden-plasmas [19,20]. Samsonov and Goree [19] observed that as dust particles in a sputtering plasma grew in diameter, the void was developed by a sudden onset of a filamentary mode in which the ionization rate and dust number density were both modulated. On the other hand, in their microgravity experiment (where dust particles are already sufficiently large) Morfill et al. [20] observed centimeter-size stable dust voids occurred without any initial turbulent phase. Experimental observations [18–20] thus reveal that a dusty plasma is not always composed of a homogeneous distribution of dust particles, but under some conditions, it is accompanied by dust free regions (dust voids).

A number of theoretical models have been proposed to explain the physics of the void formation. Goree et al. [21] theoretically explained the dust void formation by the balance of the electrostatic and ion drag forces on a dust particle. The balance of the electrostatic and ion drag forces involves electron deple-
tion (reduction of the electron number density in a dust cloud due to absorption on dust particles) and electron-impact ionization. The electron-impact ionization rate is therefore decreased in a dust cloud. However, in a void the comparatively higher ionization rate leads to an electric field that is directed outward from the void center. This gives rise to an outward ion flow that exerts an outward ion drag force on the dust particles. Therefore, in equilibrium there is a balance of forces on a dust particle: an inward electrostatic force and an outward ion drag force. Morfill et al. [20] predicted that the thermophoretic force (a neutral temperature gradient force) which is more dominant than all other forces on a dust particle in the absence of gravity, is responsible for the formation of voids.

Recently, Akdim and Goedheer [22] have shown by their simulation studies that the ion–neutral collisions increase the gas temperature by a maximum of 1 K and that the thermophoretic force or the ion drag force cannot explain the appearance of the void. On the other hand, in a recent microgravity experiment [23] the voids have been observed even in the absence of dust particles. This experimental observation also indicates that the thermophoretic force or the ion drag force acting on a dust particle cannot explain the formation of voids. In this Letter we present a theoretical model to explain the formation of plasma voids (holes) in the absence of dust particles as well as to explain the role of dust particles on the size of the voids. Specifically, we propose here that the plasma voids are self-consistent solutions of Poisson’s equation in which electrons follow the Boltzmann distribution and ions obey a vortex-like distribution because of their trapping in large amplitude plasma potentials. We have shown that the voids shrink when the dust grains are added into the plasma, and they tend to disappear if the dust number density is sufficiently high.

We consider a one-dimensional, collisionless, unmagnetized dusty plasma consisting of electrons following a Boltzmann distribution, ions obeying a trapped/vortex-like distribution, and negatively charged dust particles providing the stationary background. The electrostatic potential associated with the charge density perturbation is described by Poisson’s equation

\[ \frac{\partial^2 \phi}{\partial \xi^2} = n_e - \alpha n_i + \alpha - 1, \quad (1) \]

where \( n_e \) (\( n_i \)) is the electron (ion) number density normalized by its equilibrium value \( n_{e0} \) (\( n_{i0} \)), \( \phi \) is the electrostatic plasma potential normalized by \( k_B T_i/e \) (\( T_i \) is the ion temperature, \( k_B \) is the Boltzmann constant, and \( e \) is the magnitude of the electron charge), \( \xi \) is the space variable normalized by \( \lambda_{Dm} = (k_B T_i/4\pi n_{e0} e^2)^{1/2} \), and \( \alpha = n_{i0}/n_{e0} \). We note that \( \alpha = 1 \) represents no dust particle present in the plasma, but \( \alpha > 1 \) represents the presence of negatively charged dust particles in the background. Thus, Eq. (1) is valid in a plasma with a uniform distribution of dust particles.

The number density of electrons, which follow the Boltzmann distribution, is given by

\[ n_e = \exp(\sigma \phi), \quad (2) \]

where \( \sigma = T_i/T_e \).

To model the ion distribution in the presence of trapped particles, we employ a vortex-like ion distribution of Schamel [24] which solves the ion Vlasov equation [25–27]. Thus, we have \( f_i = f_{jf} + f_{fit} \), where

\[ f_{jf} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{|v - \beta \phi + u_0^2|}{2 \phi}\right)^2\right), \quad \text{for } |v| > \sqrt{-2\phi}, \quad (3) \]

\[ f_{fit} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\beta (v^2 + 2\phi) + u_0^2\right)\right), \quad \text{for } |v| \leq \sqrt{-2\phi}. \quad (4) \]

Here \( v \) (\( u_0 \)) is the ion (the frame) speed normalized by the ion thermal speed \( v_{Ti} = (k_B T_i/m_i)^{1/2} \), and \( |\beta| = (T_e/T_i) \), which is the ratio of the free ion temperature \( T_i \) to the trapped ion temperature \( T_{it} \), is a parameter determining the number of trapped ions. We note that the ion distribution function, as prescribed above, is continuous in velocity space and satisfies the regularity requirements for an admissible Bernstein–Greene–Kruskal (BGK) solution [24–27].

It is obvious from Eqs. (3) and (4) that \( \beta = 1 \) (\( \beta = 0 \)) represents a Maxwellian (flat-topped) distribution, whereas \( \beta < 0 \) represents a vortex-like excavated trapped ion distribution corresponding to an under-population of trapped ions. The situation \( \beta < 0 \) is of our present interest. Thus, integrating the ion distributions over the velocity space, the ion number den-
sity \( n_i \) for \( \beta < 0 \) can be expressed as \([24–26]\)

\[
n_i = \exp\left(-\frac{u_0^2}{2}\right) \left[I(\phi) + K\left(\frac{u_0^2}{2}, -\phi\right)\right] + \frac{2}{\sqrt{\pi} |\beta|} W_D(\sqrt{\beta \phi}) \tag{5}
\]

where

\[
I(x) = \left[1 - \text{erf}(\sqrt{x})\right] \exp(x), \\
W_D(x) = \exp(-x^2) \int_0^x \exp(y^2) dy, \\
K(x, y) = \frac{2}{\sqrt{\pi}} \int_0^\frac{\pi}{\sqrt{x}} d\theta \sqrt{x} \cos \theta \times \exp(-y \tan^2 \theta + x \cos^2 \theta) \times \text{erf}(\sqrt{x} \cos \theta). 
\]

For \( \phi \ll 1 \) Eq. (5) gives

\[
n_i = 1 + \frac{1}{2} Z'_r\left(\frac{u_0}{\sqrt{2}}\right) \phi - \frac{4(1 - \beta - u_0^2)}{3 \sqrt{\pi}} \exp\left(-\frac{u_0^2}{2}\right) (-\phi)^{3/2} + \frac{1}{2} \phi^2 + \cdots. \tag{9}
\]

where \( Z'_r \) represents the real part of the derivative of the dispersion function \( Z(u_0/\sqrt{2}) \) with respect to its argument \( u_0/\sqrt{2} \). We note that \( Z'_r(0) = -2 \) (corresponding to the minimum value), \( Z_r(0.924) \approx 0 \) and \( Z_r(1.5) \approx 0.57 \) (corresponding to the maximum value).

We are interested here to examine the possibility for the formation of plasma voids (holes) in our dusty plasma system. Substituting Eqs. (2) and (5) into Eq. (1), multiplying it by \( d\phi/d\xi \), and integrating the resultant equation we obtain the energy law for a pseudo-particle of unit mass \([27]\), viz.

\[
\frac{1}{2} \left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0. \tag{10}
\]

Here \( V(\phi) \) is the pseudo-potential which for our purposes reads

\[
V(\phi) = (1 - \alpha)\phi + \frac{1}{\alpha} \left[1 - \exp(\sigma \phi)\right] + \alpha \exp\left(-\frac{u_0^2}{2}\right) \times \left[1 - P(-\phi, \beta) - H\left(\frac{u_0^2}{2}, 0, -\phi\right)\right]. \tag{11}
\]

where

\[
P(x, y) = I(x) + \frac{2}{\sqrt{\pi}} \left(1 - \frac{1}{y}\right) \sqrt{y} \exp\left(\frac{y - x}{y}\right), \tag{12}
\]

\[
H(x, a, b) = \int_a^b K(x, y) dy. \tag{13}
\]

We note that the integration constant in Eq. (10) has been determined by \( V(0) = 0 \). It can be shown that \( V(\phi) = dV(\phi)/d\phi = 0 \) at \( \phi = 0 \). Therefore, localized solutions of Eq. (10) exist if (i) \( V(\phi) < 0 \) for \( 0 < |\phi| < |\phi_m| \) and (ii) \( V(\phi) = 0 \) for \( \phi = \phi_m \), where \( \phi_m \) is the amplitude of the localized potentials. The first condition can be satisfied only if \( (d^2V/d\phi^2)_{\phi=0} < 0 \) so that the fixed point at the origin is unstable. The first condition can be obtained by expanding \( V(\phi) \) around \( \phi = 0 \) and by setting the quadratic term (coefficient of \( \phi^2 \)) equal to zero. Thus, the first condition becomes \( \sigma > \alpha Z'_r(u_0/\sqrt{2})/2 \), which is always satisfied for \( 0 \leq u_0 \leq 1.306 \) since \( Z'_r(z) \leq 0 \) for \( z \leq 0.924 \) and \( \sigma > 0 \). The second condition \( V(\phi_m) = 0 \) provides a relation between \( \phi_m, \alpha, \beta, \sigma \) and \( u_0 \). To investigate the properties of the plasma voids (holes), we consider two situations, first small amplitude voids (holes) and later arbitrary amplitude voids (no limitation on the amplitude).

**Small amplitude voids/holes**

To study the properties of the plasma voids (holes) analytically, we first consider a small amplitude limit \( \phi \ll 1 \) for which we can express \( V(\phi) \) as

\[
V(\phi) = -\frac{1}{2} \left[\sigma - \frac{1}{2} \alpha Z'_r\left(\frac{u_0}{\sqrt{2}}\right)\right] \phi^2 + \frac{8\alpha}{15\sqrt{2}} \exp\left(-\frac{u_0^2}{2}\right) (1 - \beta - u_0^2)(-\phi)^{3/2}. \tag{14}
\]
The variation of $|\phi_m|$ with $\beta$ for $u_0 = 1.0$ (upper plot) and with $u_0$ for $\beta = -0.9$ (middle plot). The plot at the bottom shows the variation of $1/\delta$ with $u_0$. In all of these plots $\sigma = 0.1$, $\alpha = 1$ (solid curve), $\alpha = 5$ (dotted curve) and $\alpha = 10$ (dashed curve).

Now, using Eq. (14) and the condition $V(\phi_m) = 0$, we can express the plasma void (hole) amplitude $\phi_m$ in terms of $\alpha$, $\beta$, $\sigma$, and $u_0$, i.e., we have [25]

$$\phi_m = -\left[ \frac{15 \sqrt{\pi} \exp(u_0^2/2)}{16 \alpha (1 - \beta - u_0^2)} \left( \sigma - \frac{1}{2} \alpha Z_r\left( \frac{u_0}{\sqrt{2}} \right) \right) \right]^2. \quad (15)$$

The shape of these small amplitude plasma voids (holes) can be obtained by substituting Eq. (14) into Eq. (10) and integrating once. Thus, we have

$$\phi = \phi_m \sech^4 \left( \frac{\xi}{\delta} \right), \quad (16)$$

where the width of the plasma voids (holes) is given by

$$\delta = 4 \left[ \sigma - \frac{1}{2} \alpha Z_r\left( \frac{u_0}{\sqrt{2}} \right) \right]^{-1/2}. \quad (17)$$

Eqs. (16) and (17) indicate that the small amplitude localized solution of Eq. (10) can exist if and only if $\sigma > \alpha Z_r(u_0/\sqrt{2})/2$. We have numerically analyzed Eqs. (16) and (17) to examine the possibility for the existence of small amplitude amplitude holes, and the variation of the amplitude and the width of the holes with the plasma parameters $\alpha$ and $\beta$ as well as with the speed $u_0$. The results are displayed in Fig. 1 which shows that:

(i) small amplitude holes can exist only for higher values of $\beta$ (viz. $\phi_m \leq 0.1$ for $|\beta| \geq 10$, $u_0 = 1$, and $\alpha = 1$, or 5, or 10),

(ii) the role of $u_0$ is against for the existence of small amplitude holes for $\alpha = 1$ (i.e., no dust particle is present), but it is in favor of the existence of small amplitude holes when $\alpha > 1$ (i.e., when dust particles are present),

(iii) the width $\delta$ decreases with the dust particle number density, but increases with the speed $u_0$.

**Arbitrary amplitude voids/holes**

We are interested here to investigate arbitrary amplitude standing ($u_0 = 0$) plasma voids/holes. Using (11) and $V(\phi_m) = 0$, the amplitude (depth) $\phi_m$ of the voids/holes is determined from the nonlinear relation

$$(1 - \alpha)\phi_m + \frac{1}{\sigma} \left[ 1 - \exp(\sigma \phi_m) \right] + \alpha \exp\left( -\frac{u_0^2}{2} \right) \times \left[ 1 - P(\phi_m, \beta) - H\left( \frac{u_0^2}{2}, 0, -\phi_m \right) \right] = 0. \quad (18)$$
The width $\Delta$ of these voids/holes can be related with the depth of the classical potential $|V_{\text{min}}|$ and the amplitude $\phi_m$ by the formula [25]

$$\Delta = \frac{\phi_m}{\sqrt{|V_{\text{min}}|}}.$$  \hspace{1cm} (19)

It is clear from Eqs. (18) and (19) that for an arbitrary value of the amplitude it is not possible to find a simple analytical expressions for the amplitude (depth) $\phi_m$ and the width $\Delta$. Therefore, to study the properties (amplitude and width) of arbitrary amplitude voids (holes), we have to numerically solve either Eqs. (18) and (19) or Eq. (1) (with Eqs. (2) and (5)). The advantage of solving Eq. (1) is that in addition to the amplitude (depth) and width of the voids (holes), we can observe their profiles. Accordingly, we have numerically solved Eqs. (1), (2) and (7) to examine the plasma potential profiles and associated electron and ion density depletions. The results are displayed in Fig. 2, which shows that as we increase the dust particle number density, the depth as well as the width of the voids/holes (localized potential structure, electron and ion density depletions) decrease.

To summarize, we have considered a one-dimensional, collisionless, unmagnetized plasma composed of electrons, ions, and negatively charged dust particles, and investigated the role of negatively charged dust particles on the plasma potential structures and associated electron and ion density depletions. It is found that the plasma voids are self-consistent solutions of Poisson’s equation in which the electron density response is Boltzmannian and the ion distribution is non-Maxwellian due to the ion trapping in large amplitude plasma potentials. Furthermore, we find that as the dust particle number density is increased, the depth as well as the width of the voids (holes) decrease, and after a certain value of the dust particle number density, the voids (holes) disappear. In conclusion, we stress that our theoretical results are capable of explaining the experimentally observed scenario [20,23] of voids and their modification due to the presence of charged dust grains in a plasma. Specifically, since the observed voids are steady state phenomena, interparticle collisions are unimportant. Collisions might affect the dynamics and instability of voids (plasma holes); but the fact is that stationary voids are indeed observed in real experiments [20,23], depicting the robustness of these plasma holes. Theoretical and computational studies of the void dynamics and its stability are beyond the scope of the present Letter.

**Acknowledgements**

A.A. Mamun gratefully acknowledges the financial support of the Alexander von Humboldt-Stiftung.
This work was partially supported by the International Space Science Institute at Bern (Switzerland) as well as by the Research Training Network entitled “Complex Plasmas: The Science of Laboratory Colloidal Plasmas and Mesospheric Charged Aerosols” of the Fifth Framework Programme of the European Commission through the contract No. HPRN-CT2000-00140.

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