Instability of Obliquely Propagating Electrostatic Solitary Waves in a Magnetized Nonthermal Dusty Plasma

A. A. Mamun

Department of Physics, Jahangirnagar University, Savar, Dhaka, Bangladesh

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Abstract

A theoretical investigation has been made of instability of obliquely propagating electrostatic solitary structures in a magnetized three-component dusty plasma, which consists of a negatively charged dust fluid, Boltzmann distributed electrons, and free as well as fast ions. The Zakharov–Kuznetsov equation for these electrostatic solitary structures that exist in this plasma system is derived and their three dimensional instability is studied by the small-k (long-wavelength plane wave) perturbation expansion method. The instability criterion and its growth rate depending on the magnetic field and the propagation directions of the solitary waves are discussed. The implications of these results to some space and astrophysical plasma situations are briefly mentioned.

1. Introduction

Nowadays, there has been a rapidly growing interest in the study of different types of collective processes in dusty plasmas [1–4], plasmas with extremely massive and highly charged dust grains, for its vital role in laboratory, space, and astrophysical plasma environments, such as, cometary tails, asteroid zones, planetary rings, interstellar medium, earth’s environment, etc. It has been shown both theoretically and experimentally that the presence of extremely massive and highly charged static dust grains modifies the existing plasma wave spectra [5–7], whereas the dynamics of these extremely massive and highly charged dust grains introduces new eigenmodes [8–15]. The low frequency dust-acoustic mode [8, 15], where the dust particle mass provides the inertia and the pressures of inertialless electrons and ions provide the restoring force, is one of them. Recently, motivated by these theoretical and experimental studies [8, 15], we have investigated dust-acoustic solitary structures in a dusty plasma model consisting of a negatively charged dust fluid and isothermal or non-isothermal ions [16, 17]. The present study has extended our earlier works [16, 17] to the instability of these dust-acoustic solitary structures propagating obliquely in a magnetized three-component dusty plasma which consists of a negatively charged cold dust fluid. Boltzmann distributed electrons, and free as well as fast ions which have been found to exhibit by the numerical simulation studies on linear and nonlinear properties of dust-acoustic waves [18].

The paper is organized as follows. In Section 2 the basic equations governing our plasma system is presented. The Zakharov–Kuznetsov (ZK) equation is derived by employing the reductive perturbation method in Section 3. The solitary wave solution of this ZK equation is obtained and the properties of these electrostatic solitary structures are discussed in Section 4. In Section 5 the instability criterion and the growth rate of these solitary structures are studied. Finally, a brief discussion is given in Section 6.

2. Model equations

We consider a three-component dusty plasma consisting of extremely massive, micron-sized negatively charged cold dust fluid, Boltzmann distributed electrons, and ions with fast particles in presence of an external static magnetic field ($B_0\parallel \hat{z}$, where $\hat{z}$ is a unit vector along the z-direction). Thus, at equilibrium, we have $n_0 = Z_d n_0 + n_{e0}$, where $n_{e0}$, $n_0$, and $n_0$ are the unperturbed ion, dust, and electron number densities, respectively, and $Z_d$ is the number of electrons residing on the dust grains. The dynamics of low phase velocity (lying between the ion and dust thermal velocities, viz., $v_{ed} < v_p < v_{id}$) dust-acoustic oscillations is governed by [8, 17]

$$\frac{\partial n}{\partial t} + \nabla \cdot (nu) = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = \nabla \phi - \omega_d (u \times \hat{z}), \quad (2)$$

$$\nabla^2 \phi = n + \left( \frac{\mu}{1 - \mu} \right) e^n - \left( \frac{1}{1 - \mu} \right) n_i, \quad (3)$$

where $n$ is the dust particle number density normalized to $n_0$; $u$ is the dust fluid velocity normalized to the dust-acoustic speed $C_d = (Z_d T_i/m_d)^{1/2}$ with $T_i$ being the ion-temperature (in energy units) and $m_d$ being the mass of negatively charged dust particulates; $\phi$ is the electrostatic wave potential normalized to $T_i/e$ with $e$ being the magnitude of the electron charge; $\sigma = T_i/T_d$; $\mu = n_{e0}/n_0$; $n_i$ is the ion number density normalized to $n_0$. The time and space variables are in the units of the dust plasma period $\omega_{pd}^{-1} = (m_d/4\pi n_0 Z_d^2 e^2)^{1/2}$ and the Debye length $\lambda_{pd} = (T_i/4\pi Z_d n_0 e^2)^{1/2}$, respectively. $\omega_{pd} = (Z_d eB_0/m_d)/\omega_{pd}$ is the dust cyclotron frequency normalized to $\omega_{pd}$.

To model an ion distribution with a population of fast particles we can choose the distribution as was chosen by Cairns et al. [19]. Therefore, the ion density $n_i$ in (3) is directly given by

$$n_i = \left[ 1 + \frac{4\alpha}{1 + 3\alpha} (\phi + \phi^2) \right] e^{-\phi}, \quad (4)$$

where $\alpha$ is a parameter determining the number of fast ions [19, 20].
3. Derivation of Zakharov–Kuznetsov equation

We now follow the reductive perturbation technique and construct a weakly nonlinear theory for electrostatic waves with small but finite amplitude, which leads to a scaling of the independent variables through the stretched coordinates [21, 22]

\[
x' = \varepsilon^{1/2} x, \quad y' = \varepsilon^{1/2} y, \quad z' = \varepsilon^{1/2} (z - \nu_0 t), \quad t' = \varepsilon^{3/2} t,
\]

(5)

where \( \varepsilon \) is a small parameter measuring the weakness of the dispersion, \( \nu_0 \) is the unknown wave phase velocity (to be determined later). It may be noted here that \( x', y', z' \) are all normalized to the Debye length \( \lambda_d = \varepsilon^{-1/2} \), \( \nu_0 \) is normalized to the dust-acoustic speed \( C_A \). We can now expand the perturbed quantities about their equilibrium values in powers of \( \varepsilon \) as [21, 22]

\[
n = 1 + \varepsilon n^{(1)} + \varepsilon^2 n^{(2)} + \varepsilon^3 n^{(3)} + \cdots,
\]

\[
\phi = \phi^{(1)} + \varepsilon \phi^{(2)} + \varepsilon^2 \phi^{(3)} + \cdots,
\]

\[
u_x = \nu_x^{(1)} + \varepsilon \nu_x^{(2)} + \varepsilon^2 \nu_x^{(3)} + \cdots,
\]

\[
u_y = \nu_y^{(1)} + \varepsilon \nu_y^{(2)} + \varepsilon^2 \nu_y^{(3)} + \cdots,
\]

\[
u_z = \nu_z^{(1)} + \varepsilon \nu_z^{(2)} + \varepsilon^2 \nu_z^{(3)} + \cdots.
\]

(6)

We now use (4)–(6) in (1)–(3) and develop equations in various powers of \( \varepsilon \). To lowest order in \( \varepsilon \), i.e., equating the coefficient of \( \varepsilon \) one can obtain the first order continuity equation, \( x \)-, \( y \)-, and \( z \)-components of the momentum equation, and Poisson’s equation which in turn give

\[
\nu^{(1)} = \frac{1}{\nu_0} \nu_z^{(1)} = - \frac{1}{\nu_0} \phi^{(1)},
\]

\[
\mu^{(1)} = - \left[ \frac{\sigma \mu}{1 - \mu} + \frac{1}{1 - \mu} \left( \frac{1 - \alpha}{1 + 3\alpha} \right) \right] \phi^{(1)},
\]

\[
u_x^{(1)} = - \frac{1}{\omega_{kd}} \frac{\partial \phi^{(1)}}{\partial z}, \quad \nu_y^{(1)} = \frac{1}{\omega_{kd}} \frac{\partial \phi^{(1)}}{\partial x}.
\]

(7)

Here, the first two equations give the linear dispersion relation

\[
v_0 = \left| \frac{1}{\sigma \mu + (1 - \sigma)(1 + 3\alpha)} \right|
\]

and the last two, respectively, represent the \( x \)- and \( y \)-components of \((V_x + V_0)\), where \( V_x \) and \( V_0 \) are \( E \times B \) and diamagnetic drifts, respectively. These last two equations are also satisfied by the next higher (second) order continuity equation. Similarly, to the next higher order of \( \varepsilon \) we obtain the second order \( x \)- and \( y \)-components of the momentum equation and Poisson’s equation as

\[
\nu_x^{(2)} = - \frac{v_0}{\omega_{kd}^2} \frac{\partial^2 \phi^{(1)}}{\partial x^2} + \nu_y^{(2)} = - \frac{v_0}{\omega_{kd}^2} \frac{\partial^2 \phi^{(1)}}{\partial y^2} + \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \nu_x^{(1)} + \nu_y^{(1)}
\]

\[
\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \phi^{(1)} = n^{(2)} + \left[ \frac{\sigma \mu}{1 - \mu} + \frac{1}{1 - \mu} \left( \frac{1 - \alpha}{1 + 3\alpha} \right) \right] \phi^{(2)}
\]

\[
- \frac{1}{2} \left( \frac{1 - \sigma^2 \mu}{1 - \mu} \right) [\phi^{(1)}]^2.
\]

(8)

The first two, respectively, denote the \( x \)- and \( y \)-components of ion polarization drifts. Again, following the same procedure one can obtain the next higher order continuity equation and \( z \)-component of the momentum equation as

\[
\frac{\partial n^{(1)}}{\partial t'} - v_0 \frac{\partial n^{(2)}}{\partial z'} + \nu_x^{(2)} + \nu_y^{(2)} = 0,
\]

\[
+ \frac{\partial}{\partial z'} (u_x^{(2)} + n^{(1)} u_y^{(1)}) = 0,
\]

\[
\nu_z^{(1)} = - \frac{v_0}{\omega_{kd}} \frac{\partial u_x^{(1)}}{\partial t'} + \nu_y^{(2)} + \frac{\partial u_y^{(1)}}{\partial t'} - \frac{\partial \phi^{(2)}}{\partial z'} = 0.
\]

(9)

Now, using (7)–(9) we can readily obtain

\[
\frac{\partial \phi^{(1)}}{\partial t'} + A B \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial z'} + \frac{1}{2} A \frac{\partial}{\partial z'} \left[ \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} \right] \phi^{(1)} = 0,
\]

(10)

where

\[
A = \left[ \frac{\sigma \mu}{1 - \mu} + \frac{1}{1 - \mu} \left( \frac{1 - \alpha}{1 + 3\alpha} \right) \right]^{-3/2},
\]

\[
B = \frac{1 - \sigma^2 \mu}{1 - \mu} - 3 \left[ \frac{\sigma \mu}{1 - \mu} + \frac{1}{1 - \mu} \left( \frac{1 - \alpha}{1 + 3\alpha} \right) \right]^2,
\]

\[
D = \left( 1 + \frac{1}{\omega_{kd}^2} \right).
\]

(11)

This equation (10) is known as the Zakharov–Kuznetsov (ZK) equation or the Korteweg–de Vries (KdV) equation in three dimensions.

4. Solitary wave solution of the ZK equation

To study the solitary waves propagating in a direction making an angle \( \delta \) with the \( z' \)-axis, i.e., with the external magnetic field and lying in the \((z', x')\) plane we first rotate the coordinate axes \((x', z')\) through an angle \( \delta \), keeping the \( y' \)-axis fixed. Thus, we transform our independent variables to

\[
\xi = x' \cos \delta - z' \sin \delta, \quad n = y', \quad \eta = x' \sin \delta + z' \cos \delta, \quad t' = t.
\]

(12)

This transformation of these independent variables makes us write the ZK equation in the form

\[
\frac{\partial \phi^{(1)}}{\partial t} + \frac{\partial \phi^{(1)}}{\partial z} + \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + \frac{\partial^3 \phi^{(1)}}{\partial \xi^2 \partial \eta} + \frac{\partial^3 \phi^{(1)}}{\partial \xi \partial \eta^2} + \frac{\partial^3 \phi^{(1)}}{\partial \eta^3} = 0.
\]

(13)

where

\[
\delta_1 = A B \cos \delta,
\]

\[
\delta_2 = \frac{1}{2} A (\cos^3 \delta + D \sin^2 \delta \cos \delta),
\]

\[
\delta_3 = - A B \sin \delta,
\]

\[
\delta_4 = - \frac{1}{2} A (\sin^3 \delta + D \cos^2 \delta \sin \delta),
\]

\[
\delta_5 = A [D (\sin \delta \cos^2 \delta - \frac{1}{2} \sin^4 \delta) - \frac{1}{2} \cos^2 \delta \sin \delta],
\]

\[
\delta_6 = - A [D (\sin^2 \delta \cos \delta - \frac{1}{2} \cos^3 \delta - \frac{1}{2} \cos \delta \sin^2 \delta)],
\]

\[
\delta_7 = \frac{1}{2} A D \cos \delta,
\]

\[
\delta_8 = - \frac{1}{2} A D \sin \delta.
\]

(14)
We now look for a steady state solution of this ZK equation in the form
\[
\phi^{(1)} = \phi_0(Z),
\]
where
\[
Z = \xi - u_0 t,
\]
in which \(u_0\) is a constant velocity normalized to the dust-acoustic speed \((C_d)\). Using this transformation we can write this ZK equation in steady state form as
\[
-u_0 \frac{d\phi_0}{dZ} + \delta_1 \phi_0 \frac{d^2\phi_0}{dZ^2} + \delta_2 \frac{d^3\phi_0}{dZ^3} = 0.
\]
Now, using the appropriate boundary conditions, viz., \(\phi^{(1)} \to 0, (d\phi^{(1)}/dZ) \to 0, (d^2\phi^{(1)}/dZ^2) \to 0\) at \(Z \to \pm \infty\), the solution of this equation is given by
\[
\phi_0(Z) = \phi_{om} \text{sech}^2 \mu Z,
\]
where \(\phi_{om} = 3u_0/\delta_1\) is the amplitude and \(\mu = \sqrt{u_0^2/4\delta_1}\) is the inverse of the width of the solitary waves. It is clear from (11) and (14) that as \(\mu \) and \(\alpha\) are always less than 1, i.e., \(\lambda > 0\), depending on whether \(B\) is positive or negative the solitary waves will be either compressive \((\phi_{om} < 0)\) or rarefactive \((\phi_{om} > 0)\). Therefore, there exists solitary waves with negative potential (which are compressive solitary waves) when \(B < 0\), i.e., \(\alpha < (\sqrt{3} - 1)/(\sqrt{3} + 3) \approx 0.155\) and solitary waves with positive potential (which are rarefactive solitary waves) when \(B > 0\), i.e., \(\alpha > (\sqrt{3} - 1)/(\sqrt{3} + 3) \approx 0.155\). It is also shown from (14) and (17) that the amplitude of both the compressive and rarefactive solitary waves is inversely proportional to \(\cos \delta\). The magnitude of the external magnetic field has no direct effect on the amplitude of these solitary waves. However, it does have a direct effect on the width of these solitary waves. Figure 1 shows how the width \(1/\mu\) of these solitary waves changes with \(\delta\) and \(\omega_{ed}\).

This shows that the width of both the compressive and rarefactive solitary wave increases with \(\delta\) for its lower range, but decreases for its higher range. It is obvious that for large angles the width goes to 0 and the amplitude goes to \(\infty\). It is likely that for large angles the assumption that the waves are electrostatic is no longer valid, and we should look for fully electromagnetic structures.

5. Instability analysis

We now study the instability of the obliquely propagating solitary waves, discussed in the previous section, by the method of small-\(k\) perturbation expansion \([23-25]\). We first assume that
\[
\phi^{(1)} = \phi_0(Z) + \phi(Z, \xi, \tau, \phi),
\]
where \(\phi_0\) is defined by (17) and \(\phi\), for a long-wavelength plane wave perturbation in a direction with direction cosines \(l_i, l_j, l_k\), is given by
\[
\phi = \psi(Z) e^{i(\xi \xi + \xi_k(Z) - \omega \tau)},
\]
in which \(l_i^2 + l_j^2 + l_k^2 = 1\) and, \(\psi(Z)\) and \(\omega\), for small \(k\), can be expanded as \([23-25]\)
\[
\psi(Z) = \psi(Z) + k^2 \psi_1(Z) + k^2 \psi_2(Z) + \cdots,
\]
\[
\omega = 0 + k\omega_1 + k^2\omega_2 + \cdots.
\]
Now, substituting (20) into (13) and linearizing with respect to \(\phi\), we can express the linearized ZK equation in the form
\[
\frac{\partial \phi}{\partial \tau} - u_0 \frac{\partial \phi}{\partial Z} + \delta_1 \phi \frac{\partial^2 \phi}{\partial Z^2} + \delta_2 \frac{\partial^3 \phi}{\partial Z^3} + \delta_3 \phi \frac{\partial^2 \phi}{\partial \xi^2} + \delta_4 \frac{\partial^3 \phi}{\partial \xi^3} + \delta_5 \frac{\partial^3 \phi}{\partial Z^2 \partial \xi} + \delta_6 \frac{\partial^3 \phi}{\partial Z \partial \xi^2} + \delta_7 \frac{\partial^3 \phi}{\partial Z^2 \partial \eta} + \delta_8 \frac{\partial^3 \phi}{\partial Z \partial \eta^2} = 0.
\]
Our main object is now to find \(\omega_1\) by solving zeroth-, first- and second-order equations obtained from these last three equations. The zeroth-order equation obtained from (19)–(21) can be written, after integration, as
\[
(-u_0 + \delta_1 \phi_0)\psi_0 + \delta_2 \frac{d^2 \psi_0}{dZ^2} = C,
\]
where \(C\) is an integration constant. It is clear from (16) that the homogeneous part of this equation has two linearly independent solutions, namely
\[
f = \frac{d \phi_0}{dZ},
\]
\[
g = f \int_0^Z dZ / f^2.
\]
Therefore, the general solution of this zeroth order equation can be written as
\[
\psi_0 = C_1 f + C_2 g - Cf \int_0^Z \frac{g}{W} dZ + Cg \int_0^Z \frac{f}{W} dZ,
\]
where \(C_1\) and \(C_2\) are two constants and \(W\) is the Wronskian defined by \(W = f(dg/dZ) - g(df/dZ)\). Now, evaluating all integrals the general solution of this zeroth-order equation, for \(\psi_0\) not tending to \(\pm \infty\) as \(Z \to \pm \infty\), can finally be
The second-order equation, i.e., equation with terms involving $k$ of the first-order equation, for not tending to zero, can be written as

\[ \psi_0 = C_1 f. \]  

(26)

The first-order equation, i.e., equation with terms linear in $k$, obtained from (19)–(21) and (26), can be expressed, after integration, as

\[ (-u_0 + \delta_1 \phi_0) \psi_1 + \delta_2 \frac{d^2 \psi_1}{dZ^2} = iC_1 (a_1 + \beta_1 \tan^2 \mu Z) \phi_0 + K, \]

(27)

where $K$ is the integration constant and $a_1$ and $\beta_1$ are given by

\[
\begin{align*}
\alpha_1 &= (a_1 + l_z u_0) - \frac{1}{2} \phi_{0m} \mu_1 - 2 \mu_2 \\
\beta_1 &= \frac{1}{2} \phi_{0m} \mu_1 - 6 \mu_2 \\
\mu_1 &= \delta_1 l_z + \delta_1 l_z \\
\mu_2 &= 3 \delta_1 l_z + \delta_3 l_z.
\end{align*}
\]

(28)

Now, following the same procedure the general solution of this first-order equation, for $\psi_1$ not tending to $\pm \infty$ as $Z \to \pm \infty$, can be expressed as

\[ \psi_1 = K_1 f + \frac{iC_1}{8 \delta_2 \mu^2} [(a_1 + \beta_1) Z f + \frac{2}{3} (3 \alpha_1 + \beta_1) \phi_0]. \]

(29)

The second-order equation, i.e., equation with terms involving $k^2$, obtained from (19)–(21), can be written as

\[
\begin{align*}
- u_0 \frac{d}{dZ} + \delta_1 \frac{d}{dZ} \phi_0 + \delta_2 \frac{d^3}{dZ^3} \psi_2 &= i \omega_2 \psi_0 + i (\alpha_1 + l_z u_0) \psi_1 - i \mu_1 \phi_0 \psi_1 \\
&+ \mu_3 \frac{d^2 \psi_1}{dZ^2} - i \mu_2 \frac{d^4 \psi_1}{dZ^4},
\end{align*}
\]

(30)

where

\[ \mu_3 = 3 \delta_2 \mu^2 + 2 \delta_3 l_z + \delta_5 l_z^2 + \delta_7 l_z^2. \]

(31)

The solution of this second-order equation exists, if the right-hand side is orthogonal to a kernel of the operator

\[
- u_0 \frac{d}{dZ} + \delta_1 \frac{d}{dZ} \phi_0 + \delta_2 \frac{d^3}{dZ^3}.
\]

(32)

This kernel, which must tend to zero as $Z \to \pm \infty$, is $\phi_0/\phi_{0m} = \text{sech}^2 \mu Z$. Thus, we can write the following equation determining $\omega_1$:

\[
\int_{-\infty}^{\infty} \phi_0 \left[ i \omega_2 \psi_0 + i (\alpha_1 + l_z u_0) \psi_1 - i \mu_1 \phi_0 \psi_1 \\
+ \mu_3 \frac{d^2 \psi_1}{dZ^2} - i \mu_2 \frac{d^4 \psi_1}{dZ^4} \right] dZ = 0.
\]

(33)

Now, substituting the expressions for $\psi_0$ and $\psi_1$ given, respectively, by (26) and (29) and then performing the integration we arrive at the following dispersion relation:

\[ \omega_1 = \Omega - l_z u_0 + \sqrt{\Omega^2 - \gamma}; \]

(34)

where

\[ \Omega = \frac{2}{3} (\phi_{0m} \mu_1 - 2 \mu_2 \mu^2), \]

\[ \gamma = \frac{1}{3 \sqrt{2}} (\phi_{0m} \mu_1 - 3 \phi_{0m} \mu_1 \mu_2 \mu^2 - 3 \mu_2^2 \mu^4 + 12 \mu_2 \mu^4). \]

(35)

It is clear from our dispersion relation (34) that there is always instability if $\gamma - \Omega^2 > 0$. Thus, using (11), (14), (28), (31) and (35) we can express this instability criterion as

\[ S_1 > 0, \]

\[ S_1 = \sin^2 \delta - \frac{5 l_x^2}{3 l_n} \tan^2 \delta + \omega_{cd}^2 \left( 1 + \frac{l_z^2}{l_n^2} - \frac{5 l_x^2}{3 l_n} \tan^2 \delta \right) \]

(36)

and the growth rate $\gamma$ of this instability as

\[ \gamma = \frac{2}{\sqrt{135}} \frac{u_0 l_z \omega_{cd}}{\omega_{cd}^2 + \sin^2 \delta \sqrt{S_1 (1 + \omega_{cd}^2)}.} \]

(37)

Now, differentiating $S_1$ with respect to $\omega_{cd}^2$ one can obtain

\[ \frac{\partial S_1}{\partial \omega_{cd}^2} = 1 + \frac{l_z^2}{l_n} \left( 1 - \frac{5}{3} \tan^2 \delta \right) \]

\[ = 1 + \frac{8 l_z^2}{3 l_n^2} \cos^2 \delta - \frac{5 l_x^2}{3 l_n^2}. \]

(38)

This means that if $[1 + (l_z^2/l_n^2)] > (\leq) \frac{5}{3} \tan^2 \delta$, the instability, i.e., the growth rate of the instability increases (decreases) with $\omega_{cd}^2$. The condition $S_1 = 0$, which expresses the threshold value of $\omega_{cd}$ in terms of $l_z/l_n$ and $\delta$, can be written as

\[ \omega_{cd}^2 = \left( \frac{\cos^2 \delta - 1}{\cos^2 \delta - \frac{5 l_x^2}{3 l_n^2}} \right) \left( 1 + \frac{8 l_z^2}{3 l_n^2} \cos^2 \delta - \frac{5 l_x^2}{3 l_n^2} \right). \]

(39)

Therefore, $\omega_{cd}^2$ has a resonance at

\[ \delta_w = \cos^{-1} \left[ \frac{5 l_z^2}{3 l_n^2} \left( 1 + \frac{8 l_z^2}{3 l_n^2} \cos^2 \delta - \frac{5 l_x^2}{3 l_n^2} \right) \right]. \]

(40)
and zero at
\[ \delta = 0 \quad \text{when} \quad \frac{l_f}{l_q} \geq \frac{\sqrt{3}}{\sqrt{5}}. \]
\[ \delta_0 = \cos^{-1} \left( \frac{5 l_f^2}{3 l_q^2} \right) \quad \text{when} \quad \frac{l_f}{l_q} < \frac{\sqrt{3}}{\sqrt{5}}. \]  \( (41) \)

We note from (38) that for \( \delta < \delta_0, \delta S_\delta/\delta \omega^2_\delta > 0 \) and so that the instability region lies above \( S_\delta = 1 \) and for \( \delta < \delta_0, \delta S_\delta/\delta \omega^2_\delta > 0 \) and so that the instability region lies below \( S_\delta = 1 \). Thus, we have two cases, depending on whether \( l_f/l_q \) is larger or smaller than \( \sqrt{3}/5 \). Figure 2 shows the instability regions in these two cases.

6. Discussion

The properties of electrostatic solitary structures as well as their instabilities have been investigated rigorously. We have studied the nature of these solitary structures by introducing the Zakharov–Kuznetsov (ZK) equation and then analysed the stability of these structures by small-\( k \) perturbation expansion method. The results which have been found in this investigation may be summarized as follows:

(i) The effects of nonthermal ions, magnetic field, obliqueness have been found to modify the nature of the electrostatic solitary structures in a dusty plasma. It has been shown that the presence of nonthermal/fast ions may allow compressive and rarefactive solitary waves to exist. It is found that, depending on the value of \( \alpha \), the solitary structures may change from compressive to rarefactive and that for \( \alpha < 0.155 \) there exist solitary waves with negative potential (compressive solitary waves or solitary waves with density hump) and for \( \alpha > 0.155 \) there exist solitary waves with positive potential (rarefactive solitary waves or solitary waves with density dip).

(ii) The effects of the obliqueness and the external magnetic field make these solitary structures unstable as well as change the amplitude and width of these solitary structures. It is found that the amplitude of both the compressive and rarefactive solitary waves is inversely proportional to \( \cos \delta \) (eqs. (14) and (17)). It is shown that the width of these solitary waves increases with \( \delta \) for its lower range and decreases for its higher range (Fig. 1). It should be pointed out that for large angles the assumptions that the waves are electrostatic is no longer valid, and we should look for fully electromagnetic structures.

(iii) The magnitude of the external magnetic field \( B_0 \) has no direct effect on the amplitude of these solitary waves. However, it does have a direct effect on their width. We have found that as its magnitude increases, the width of both the compressive and rarefactive solitary waves decreases, i.e., the external magnetic field makes the solitary structures more spiky.

(iv) We have shown that the \( x \)-value has no effect on whether the solitary waves will be stable or unstable. However, the stability of these structures strongly depend on the external magnetic field and the propagation directions of both the nonlinear waves and their perturbation mode.

(v) We have drawn a plot of \( S_\delta = 1 \) (Fig. 2) in parameter space \( (\omega_{cd}, \delta) \) and shown two instability regions for two different cases, namely, \( l_f/l_q < \sqrt{3} \) and \( l_f/l_q \geq \sqrt{3} \). When \( l_f/l_q = \sqrt{3}/5.1 \), the waves become unstable for \( 0^\circ < \delta < 25^\circ \) and when \( l_f/l_q = \sqrt{3}/4.9 \), the waves become unstable for \( 8^\circ < \delta < 30^\circ \). We have also shown that for \( [1 + (l_f^2/l_q^2)] > (\gamma^2) \tan^2 \delta \), the growth rate of the instability increases (decreases) with \( \omega^2_\delta \).

We have analysed the instability of these solitary structures by the reductive perturbation method and small-\( k \) perturbation expansion which are valid for small but finite amplitude solitary waves and long wavelength perturbation mode. Since in many astrophysical situations there may exist extremely large amplitude solitary waves and short wavelength perturbation mode, we propose to develop a more exact theory for stability analysis of arbitrary-amplitude solitary waves and arbitrary wavelength perturbation modes by generalization of our present work to such waves and modes. However, our present analysis should be useful for understanding different nonlinear features of localized and unstable electrostatic disturbances in many space plasma situations in which extremely massive dust grains, thermal or nonthermal ions and free electrons are the major plasma species. This investigation should also be important in understanding coagulation or condensation of the dust grains in such a space and astrophysical dusty plasma system.

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References