Transverse mode solitons in a hot ultrarelativistic electron–positron plasma

U. A. Mofiz and A. A. Mamun

Institute of Nuclear Science and Technology, Atomic Energy Research Establishment, Savar, GPO Box No. 3787, Dhaka, Bangladesh

(Received 26 October 1992; accepted 25 January 1993)

Intense short-duration transverse mode solitons are found to exist in a ultrarelativistically hot electron–positron plasma and its relation with the pulsar radiation in discussed.

In this Brief Communication, we extend our earlier research to consider the nonlinear propagation of the transverse mode in a hot ultrarelativistic electron–positron plasma. The microscopic state of the plasma is described by the kinetic equation

\[
\frac{\partial \delta f_s}{\partial t} + \frac{c^2}{\varepsilon_s} \frac{\partial}{\partial p_s} \delta f_s + e_s \mu \frac{\partial \delta f_s}{\partial p_s} = 0,
\]

where \(\delta f_s(r,p,t)\) is the perturbation of the distribution function, \(s=(e,p)\) denotes the species electron and positron, respectively; \(\varepsilon_s=c(p_s^2 + m_s^2c^2)^{1/2}\) and \(f_{so}(p)\) is the equilibrium plasma distribution.

Considering the perturbation \(\delta f_s \sim \exp(-i\omega t + ikx)\), we obtain

\[
\delta f_s = \frac{ie_s(E \cdot v_s) \delta f_{so}}{\omega - k \cdot v_s}.
\]

From the Maxwell equation

\[
J = \sum_s \varepsilon_s \int v_s \delta f_s \, dp = \sigma(\omega,k)E,
\]

and

\[
\varepsilon_{ij}(\omega,k) = \delta_{ij} + \frac{4\pi i}{\omega} \sigma_{ij}(\omega,k),
\]

we find the dielectric function

\[
e^{\varepsilon}(\omega,k) = 1 + \sum_s \frac{2me_s^2}{\omega k^2} \int dp \frac{(k \times v)^2 \delta f_{so}}{\omega - k \cdot v}.
\]

For an ultrarelativistic \((p_s^2 \gg m^2c^2)\) plasma, we consider the local Maxwellian distribution:

\[
f_{so} = \left( \frac{n_e}{\pi \varepsilon^2} \right) e^{-\varepsilon^2/\varepsilon^2}.
\]

Then

\[
e^{\varepsilon}(\omega,k) = 1 - \frac{1}{k^2\lambda_D^2} \left[ 1 + \frac{\omega}{2kc} \frac{k^2c^2}{\omega^2} - 1 \right] \ln \left( \frac{\omega + kc}{\omega - kc} \right),
\]

with

\[
\frac{1}{\lambda_D^2} = \sum_s \frac{1}{\lambda_D^2_s}, \quad \frac{1}{\lambda_D^2} = \frac{T_s}{4\pi e_s^2 n_s}.
\]

Here \(e^{\varepsilon}(\omega,k) = k^2c^2/\omega^2\) defines the possible transverse modes in the ultrarelativistic electron–positron plasma, which are

\[
\omega^2 = \frac{1}{2} k^2c^2 \left[ 1 + \frac{3}{2k^2\lambda_D^2} \left( 1 + \frac{13}{3k^2\lambda_D^2} + \frac{9}{4k^4\lambda_D^4} \right)^{1/2} \right],
\]

\[
\omega = \frac{3c}{2\lambda_D} \left( 1 + \frac{k^2\lambda_D^2}{6} \right), \quad k^2\lambda_D^2 < 1,
\]

\[
\omega = \frac{1}{v^2} kc \left( 1 + \frac{11}{6k^2\lambda_D^2} \right), \quad k^2\lambda_D^2 > 1.
\]

We study the propagational properties of mode 1 [Eq. (9)] and mode 2 [Eq. (10)] in detail. For mode 1, we find

\[
u_{ph} = \frac{c}{k} \left( 1 + \frac{1}{6} k^2\lambda_D^2 \right) > c,
\]

\[
u_{g} = \frac{d\omega}{dk} = \frac{c(k\lambda_D)}{\sqrt{6}} < c,
\]

\[
u'_{g} = \frac{d^2\omega}{dk^2} = \frac{c\lambda_D}{6} > 0;
\]

and for mode 2,

\[
u_{ph} = \frac{c}{k} \left( 1 + \frac{11}{6k^2\lambda_D^2} \right) > c,
\]

\[
u'_{g} = \frac{d^2\omega}{dk^2} = \frac{c}{3k^2} \left( 1 - \frac{11}{6k^2\lambda_D^2} \right) < c,
\]

\[
u'_{g} = \frac{d^2\omega}{dk^2} = \frac{11}{3v^2} c(k\lambda_D^2) > 0.
\]

It shows, as the usual transverse wave the phase velocities of both modes are higher than the speed of light but...
the waves are dispersive in nature. Therefore, we study the nonlinear propagation of these modes in detail.

The nonlinear evolution of the wave is described by the nonlinear Schrödinger equation

$$i(\partial_t + v_x \partial_x)E + \frac{1}{2} k^2 \partial_x^2 E - \Delta E = 0,$$

where $\Delta = \partial \omega / \partial |E|^2$ is the nonlinear frequency shift caused by the density fluctuation $\delta n(|E|^2)$ due to the high-frequency ($\omega$) wave propagation. The slow plasma response, in this context, can easily be described by the fluid equation

$$\partial_t \gamma P + \partial_x (\gamma P \mu_x) = 0,$$

where

$$\gamma = \left(1 - \frac{v_x^2}{c^2}\right)^{-1/2}, \quad m_* = m c^2 \gamma \mu_x, \quad \mu_x = n_0 T_s,$$

and

$$f_p = \frac{\phi}{c} \left( u_x \cdot B \right) = -m_* c^2 \partial_x \sqrt{1 + \beta^2 |E|^2},$$

with $\beta_x = e_x / m_* c^2 c^2 \omega^2$ the ponderomotive force due to the high-frequency transverse wave.

We consider the isothermal state of plasma ($T_e = T_p = T$) and also consider $\gamma_{pe} = \gamma_0$. In this case ponderomotive force (20) is charge independent and we can neglect the ambipolar field $\phi$ in (19).

Considering $n = n_0 + \delta n$, $T = T_0 + \delta T$ ($\delta n \ll n_0$, $\delta T \ll T_0$) from adiabatic law for ultrarelativistic gas ($n/T_0^4 = \text{const}$), we find $\delta T/T_0 = \delta n/n_0$ and $\delta \rho = \nu_{p0} \delta n$. Then from the system of Eqs. (18) and (19) we obtain

$$\left(\partial_t^2 - v_x^2 \partial_x^2\right) \frac{\delta n}{n_0} = \frac{c^2}{\gamma_0} \partial_x \sqrt{1 + \beta |E|^2},$$

$$\partial_t^2 \frac{4 T_0}{3 m_* \gamma_0},$$

which in the moving frame $\xi = x - v_x t$ gives,

$$\frac{\delta n}{n_0} = \frac{c^2}{\gamma_0 (v_x^2 - v_y^2)} \left(\sqrt{1 + \beta |E|^2} - 1\right).$$

Thus the evolution equation of the wave takes the form

$$i(\partial_t + v_p \partial_x)E + \frac{1}{2} k^2 \partial_x^2 E + Q |E|^2 E = 0,$$

with

$$Q = \frac{v_p^2 \beta}{k \lambda_0^2 \gamma_0 (v_x^2 - v_y^2)}.$$