Low-frequency electrostatic dust-modes in a strongly coupled dusty plasma with dust charge fluctuations

A. A. Mamun,[a] P. K. Shukla, and T. Farid
Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany

(Received 17 January 2000; accepted 24 February 2000)

Low-frequency electrostatic dust-modes are theoretically investigated accounting for dust grain charge fluctuation and for equilibrium grain charge inhomogeneity in a strongly coupled dusty plasma. A new stable extremely low-frequency mode, which is due to the inhomogeneity in the equilibrium dust grain charge, is found to exist in such a dusty plasma. It is also found that the dust-acoustic mode becomes unstable due to the effect of this equilibrium dust grain charge inhomogeneity. It is observed here that the influence of strong correlations in the dust fluid significantly modify the dispersion properties of this new mode as well as of the existing dust-acoustic mode. The implications of our results to recent experimental observations and to some space and astrophysical situations are briefly discussed. © *2000 American Institute of Physics.* [S1070-664X(00)02606-9]

I. INTRODUCTION

There has been a rapidly growing interest in understanding the physics of strongly coupled dusty plasma and associated low-frequency dust-modes because of their vital role in space and astro-plasmas (such as, white dwarf matter, interior of heavy planets, etc.), laboratory plasmas (for examples, plasma crystals, plasmas produced by laser compression of matter, etc.), and industrial plasma processing. It was first pointed out by Ikezi[1] that a classical Coulomb plasma with micron-sized dust particles can readily go into the strongly coupled regime. This is because: The coupling constant $\Gamma[=(q_d^2/a_d k_B T_d)\exp(-a_d/\lambda_D)$, where q_d is dust charge, a_d is the inter grain distance, λ_D is the dusty plasma Debye length, k_B is the Boltzmann's constant, and T_d is the dust temperature] for a micron size dust grain can easily be order of unity or larger. Recent laboratory experiments[2–4] as well as a number of theoretical analyses[5–8] conclusively verified the prediction of Ikezi[1] and demonstrated that the dust particles organize themselves into crystalline patterns in such a dusty plasma. It is also observed by experiments that as the temperature of the dust is increased (i.e., Γ is decreased), the dust crystals melt and then vaporize so that one encounters the usual weakly coupled ideal Coulomb plasma. Thus, laboratory experiments in such a dusty plasma system provide an excellent opportunity for the study of transitions from the strongly coupled to weekly coupled regimes.

One of the most important characteristics of such a dusty plasma system is the fact that the charge on a dust particle is not constant, but may change in time. This is basically a quite common feature of a dusty plasma. The charge can fluctuate in two ways: First, it varies randomly due to the collection of individual electrons and ions at random times; second, in the presence of perturbations it varies when a particle moves in an inhomogeneous plasma or a plasma that varies with time. Recently, Nunomura *et al.*[9] have performed an experiment based on the latter mechanism and have observed a self-excited vertical oscillation of dust particles, which are proposed to arise from the charge deviation from its equilibrium value, due to the delay in charging. Motivated by these experimental observations,[9] we have considered a dusty plasma system consisting of Boltzmann distributed electrons and ions, and strongly correlated dust fluid where the charge on a dust particle fluctuates and its equilibrium value also varies with space. We are interested here to investigate whether effects of the dust grain charge inhomogeneity or strong correlations in the dust fluid introduce new eigenmodes and instabilities, or just modify the other existing dust-modes. It is found here that the effect of the inhomogeneity in the equilibrium dust grain charge introduces an extremely low-frequency new stable mode as well as it causes the dust-acoustic mode to become unstable.

The manuscript is organized as follows. We presented our dusty plasma model equations in Sec. II. We derived the dispersion relation and analyzed numerically the dispersion properties of the new mode as well as the dust-acoustic mode in Sec. III. Finally, a brief discussion is presented in Sec. IV.

II. GOVERNING EQUATIONS

We consider a three component unmagnetized dusty plasma consisting of electrons, ions, and strongly coupled dust grains whose charge varies due to dust charge gradient and fluctuating currents flowing onto the dust grain surface. We assume that dust grains are strongly coupled because of their lower temperature and larger electric charge, whereas electrons and ions are very weakly coupled due to their higher temperatures and smaller electric charges. Thus, on the extremely slow dust time scale, the electrons and ions are

[a]Also at: Department of Physics, Jahangirnagar University, Savar, Dhaka, Bangladesh. Electronic mail: am@egal.tp4.ruhr-uni-bochum.de

 © 2000 American Institute of Physics

Downloaded 14 Feb 2001 to 130.239.166.30. Redistribution subject to AIP copyright, see http://ojps.aip.org/pop/popcpyrts.html

in local thermodynamic equilibrium, and their number densities, N_e and N_i, obey the Boltzmann distributions,[10,11] namely

$$N_e = n_{e0} \exp\left(\frac{e\Phi}{k_B T_e}\right) \tag{1}$$

and

$$N_i = n_{i0} \exp\left(-\frac{e\Phi}{k_B T_i}\right), \tag{2}$$

where n_{e0} (n_{i0}) and T_e (T_i) are the unperturbed number density and the temperature of electrons (ions), respectively, Φ is the electrostatic wave potential, and e is the magnitude of the electron charge.

The dust particles are assumed to be spherical with the same radius r_d and the surface charge q_d. If the dust particles are charged due to the collection of the electrons and ions from the background plasma, we have for the electron (ion) current I_e (I_i)[12–16]

$$I_e(q_d,\Phi) = -\pi r_d^2 e\left(\frac{8k_B T_e}{\pi m_e}\right)^{1/2} N_e \exp\left(\frac{eq_d}{r_d k_B T_e}\right) \tag{3}$$

and

$$I_i(q_d,\Phi) = \pi r_d^2 e\left(\frac{8k_B T_i}{\pi m_i}\right)^{1/2} N_i\left(1-\frac{eq_d}{r_d k_B T_i}\right), \tag{4}$$

where m_e (m_i) is the mass of the electron (the ion). On extremely slow dust time scale, the dynamics of charge fluctuating dust oscillations is governed by the well-known generalized hydrodynamic (GH) equations,[17–19] and the charging equation,[12–16] which are

$$\frac{\partial N_d}{\partial t} + \nabla\cdot(N_d \mathbf{U_d}) = 0, \tag{5}$$

$$\left(1+\tau_m\frac{\partial}{\partial t}\right)(m_d N_d D_t \mathbf{U_d} + q_d N_d \nabla\Phi + \nabla P_d)$$

$$= \eta\nabla\cdot\nabla\mathbf{U_d} + \left(\zeta+\frac{\eta}{3}\right)\nabla(\nabla\cdot\mathbf{U_d}), \tag{6}$$

and

$$D_t q_d = I_e(q_d,\Phi) + I_i(q_d,\Phi). \tag{7}$$

The equations are closed with the help of Poisson's equation

$$\nabla^2\Phi = 4\pi[e(N_e - N_i) - q_d N_d], \tag{8}$$

where $D_t = \partial/\partial t + \mathbf{U_d}\cdot\nabla$, N_d is the dust particle number density, $\mathbf{U_d}$ is the dust fluid velocity, m_d is the dust particle mass, N_d and P_d are the dust fluid density and the pressure, τ_m is the viscoelastic relaxation time, η and ζ are transport coefficients of shear and bulk viscosities. Various approaches for calculating these transport coefficients have been widely discussed in the literature.[17–23] The viscoelastic relaxation time τ_m is given by[18,19]

$$\tau_m = \frac{m_d \eta_l}{k_B T_d}\left(1-\mu_d+\frac{4}{15}u(\Gamma)\right)^{-1}, \tag{9}$$

where μ_d is the compressibility,[18] and is defined as

$$\mu_d = \frac{1}{k_B T_d}\frac{\partial P_d}{\partial n_d} = 1+\frac{1}{3}u(\Gamma)+\frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial\Gamma}, \tag{10}$$

and $u(\Gamma)$ is a measure of the excess internal energy of the system and is calculated for weakly coupled plasmas ($\Gamma<1$) as[22] $u(\Gamma)\simeq -(\sqrt{3}/2)\Gamma^{3/2}$. To express $u(\Gamma)$ interms of Γ for a range of $1<\Gamma<100$, Slattery *et al.*[23] have analytically derived a relation

$$u(\Gamma)\simeq -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81, \tag{11}$$

where a small correction term due to finite number of particles is neglected. The dependence of the other transport coefficient $\eta_l = (\frac{4}{3})\eta' + \zeta'$ (referred to longitudinal viscosity coefficient) on Γ are somewhat more complex and cannot be expressed in such a closed analytical form, where $\eta' = \eta/(m_d n_{d0})$, $\zeta' = \zeta/(m_d n_{d0})$, with n_{d0} is the equilibrium dust particle density. However, tabulated/graphical results of their functional behavior derived from molecular-dynamic (MD) simulations and a variety of statistical schemes are available in the literature.[18] Typical values[18] of η_l are $1.04a_d^2\omega_{pd}$ for $\Gamma=1$, $0.08a_d^2\omega_{pd}$ for $\Gamma=10$, and $0.3a_d^2\omega_{pd}$ for $\Gamma=160$, where $\omega_{pd} = (4\pi n_{d0} Z_{d0}^2 e^2/m_d)^{1/2}$.

III. DISPERSION PROPERTIES

To study low-frequency electrostatic dust-modes in the strongly coupled dusty plasma system under consideration, we shall carry out a normal mode analysis. We first express our dependent variables N_d, U_d, Φ, and q_d in terms of their equilibrium and perturbed parts as

$$N_d = n_{d0} + n_d, \tag{12}$$

$$\mathbf{U_d} = 0 + \mathbf{u_d}, \tag{13}$$

$$q_d = -eZ_{d0}(x) - eZ_d, \tag{14}$$

and

$$\Phi = 0 + \phi, \tag{15}$$

where $Z_{d0}(x)$ is number of electrons residing in the dust grain at equilibrium, which is not constant, but varies with x. Now, using (1)–(4) and (12)–(15), we first linearize our basic equations, (5)–(8), to first-order approximation and express them as

$$\frac{\partial n_d}{\partial t} + n_{d0}(\nabla\cdot\mathbf{u_d}) = 0, \tag{16}$$

$$\left(1+\tau_m\frac{\partial}{\partial t}\right)\left(\frac{\partial u_d}{\partial t} - \frac{Z_{d0}e}{m_d}\nabla\Phi + \frac{\mu_d v_{td}^2}{n_{d0}}\nabla n_d\right)$$

$$= \eta'\nabla^2\mathbf{u_d} + \left(\zeta'+\frac{1}{3}\eta'\right)\nabla(\nabla\cdot\mathbf{u_d}), \tag{17}$$

$$\left(\frac{\partial}{\partial t}+\nu_1\right)Z_d + u_{dx}\left(\frac{\partial Z_{d0}}{\partial x}\right) = \nu_2\left(\frac{n_e}{n_{e0}} - \frac{n_i}{n_{i0}}\right), \tag{18}$$

and

Downloaded 14 Feb 2001 to 130.239.166.30. Redistribution subject to AIP copyright, see http://ojps.aip.org/pop/popcpyrts.html

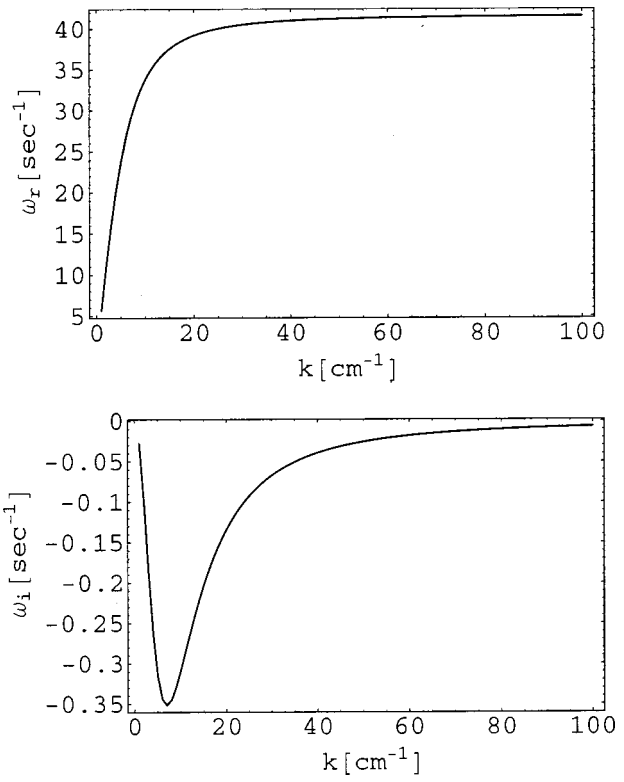

FIG. 1. The dispersion properties of the damped dust-acoustic mode for $k_d=0$: The upper plot is for ω_r and the lower one is for ω_i.

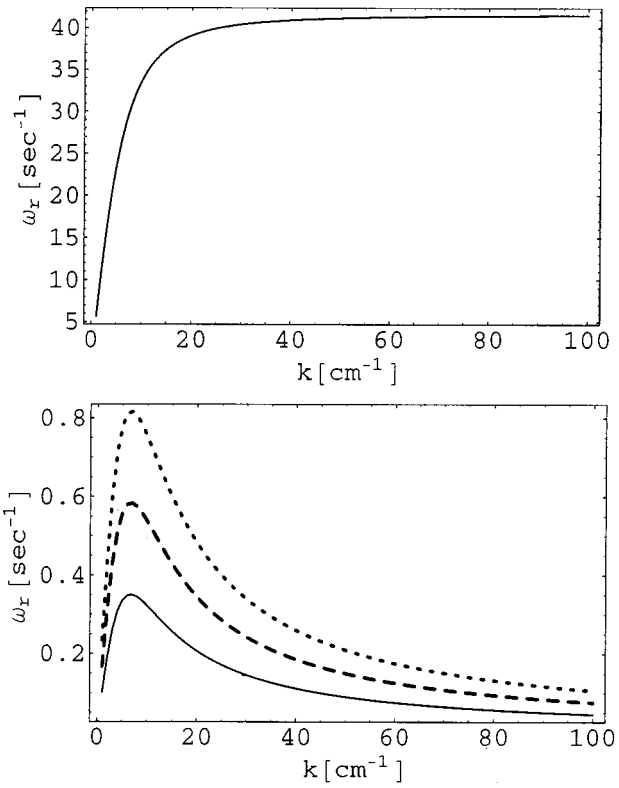

FIG. 2. The dispersion properties of the dust-acoustic mode as well as our new low-frequency stable mode (which is only due to the equilibrium dust grain charge inhomogeneity) for $k_d=0.3$ (solid curve), $k_d=0.5$ (dashed curve), and $k_d=0.7$ (dotted curve): The upper plot shows the dispersion curve for existing dust-acoustic mode, whereas the lower one is for our new mode.

$$\nabla^2\phi = 4\pi e(n_e - n_i + Z_{d0}n_d + Z_d n_{d0}), \tag{19}$$

where $v_{td}=(k_B T_d/m_d)^{1/2}$, $\nu_1=|I_{e0}|e[1/(r_d k_B T_e)+1/(r_d k_B T_i+Z_{d0}e^2)]$, $\nu_2=|I_{e0}|/e$, $I_{e0}=I_e(q_d=-Z_{d0}e,\Phi=0)$. To derive the dispersion relation, we first perform Fourier transform (i.e., use $\partial/\partial t=-i\omega$ and $\nabla=ik$) and then find n_d and Z_d.

The substitution of these first-order perturbed quantities, viz. n_d and Z_d in to the linearized Poisson's equation (19), yields the dispersion relation

$$1+\left(1+\frac{i\nu_2(n_{d0}/n_{e0})}{\omega+i\nu_1}\right)\frac{1}{k^2\lambda_{De}^2} +\left(1+\frac{i\nu_2(n_{d0}/n_{i0})}{\omega+i\nu_1}\right)\frac{1}{k^2\lambda_{Di}^2}-\omega_{pd}^2\left(1-i\frac{k_d}{k}\frac{\omega}{\omega+i\nu_1}\right) \times\left(\omega^2-\mu_d k^2 v_{td}^2+i\frac{\eta_l\omega k^2}{(1-i\omega\tau_m)}\right)^{-1}=0, \tag{20}$$

where $k_d=(1/Z_{d0})(\partial Z_{d0}/\partial x)$ is the inverse of the dust grain-charge inhomogeneity scale length. It is noted here that $k_d=0$ for homogeneous dust grain charge (i.e., for $Z_{d0}=$constant). It can be shown that if we consider the dust grains with constant and homogeneous charge (i.e., $\nu_{1,2}=0$ and $k_d=0$) and weakly coupled dusty plasma (i.e., $\Gamma\ll 1$ or $\tau_m=0$, $\mu_d=0$, and $\eta_l=0$), Eq. (20) stands for the dispersion relation for the dust-acoustic mode studied by Rao *et al.*[11] Again, if we consider a weakly coupled dusty plasma with the dust grain charge fluctuation (but the dust grain charge at equilibrium being constant, viz. $k_d=0$), (20) becomes the dispersion relation for the damped dust-acoustic waves.[12–16] Thus, our main interest here is to investigate whether effects of the dust grain charge inhomogeneity or strong correlations in the dust fluid introduce new eigenmodes and instabilities, or just modify the existing dust modes and instabilities. To examine the effects of the inhomogeniety in the equilibrium dust grain charge on low-frequency electrostatic waves in a dusty plasma with the dust grain charge fluctuation, we numerically analyze our general dispersion relation (20) for different values of k_d (such as, $k_d=0$, $k_d=0.3$, $k_d=0.5$, and $k_d=0.7$) with typical plasma parameters of interest: $T_e=10^4\,^\circ$K, $T_i=10^3\,^\circ$K, $T_d=300\,^\circ$K, $m_d=10^{-12}$ gm, $n_{e0}=10^7$ cm^{-3}, $n_{d0}=10^4$ cm^{-3}, $Z_{d0}=10^2$, $\tau_m=0$, $\mu_d=0$, and $\eta_l=0$. The results are displayed in Figs. 1–3. Figure 1, where $k_d=0$ is used, shows a damped dust-acoustic mode: The upper plot shows how ω_r changes with k and the lower one shows how damping rate (ω_i) varies with k. Figure 2, where $k_d=0.3$ (solid curve), $k_d=0.5$ (dashed curve), and $k_d=0.7$ (dotted curve), shows the dispersion properties of the dust-acoustic mode as well as a very low-frequency new stable mode which is only due to the equilibrium dust grain charge inhomogeneity: The upper plot shows the dispersion curve for the dust-acoustic mode, whereas the lower one shows those for our new mode. Figure 3 shows how the damping rate of the dust-acoustic mode and the growth rate of the other branch (corresponding to the real negative root) of this dust-acoustic mode, which has been found here to be

Downloaded 14 Feb 2001 to 130.239.166.30. Redistribution subject to AIP copyright, see http://ojps.aip.org/pop/popcpyrts.html

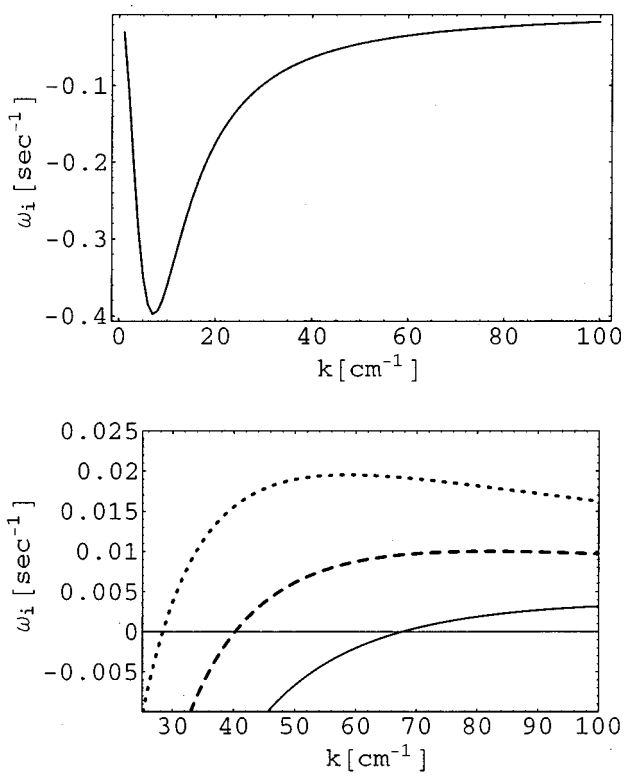

FIG. 3. The variation of the damping rate of the dust-acoustic mode (the upper plot) and the growth rate (the lower plot) of the other branch (corresponding to the real negative root) of the dust-acoustic mode for $k_d=0.3$ (solid curve), $k_d=0.5$ (dashed curve), and $k_d=0.7$ (dotted curve).

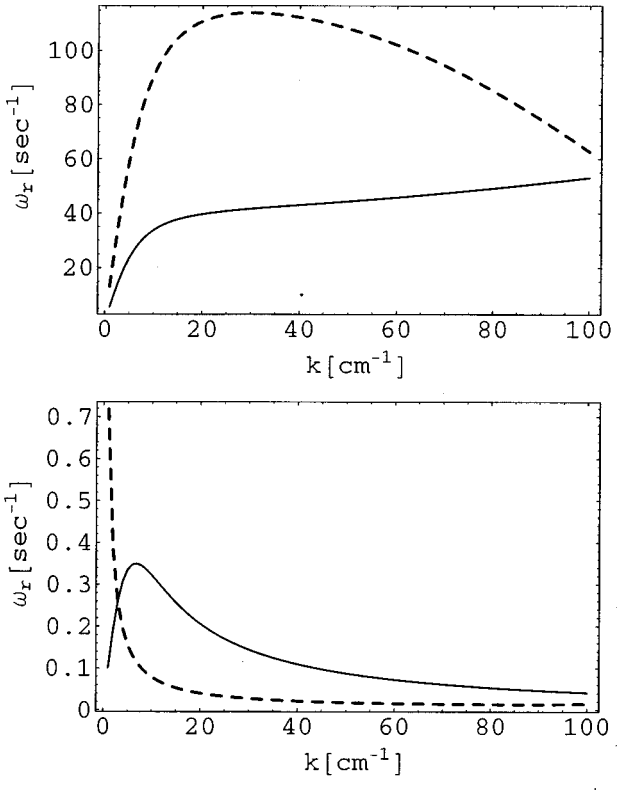

FIG. 4. The dispersion properties of the dust-acustic mode (the upper plot) and our new mode (the lower plot) are modified by strong correlations in the dust fluid (in both cases, solid curves for $\Gamma=1$ and dashed curves for $\Gamma=10$).

unstable due to the effect of the equilibrium dust grain charge inhomogeneity, changes with k_d: $k_d=0.3$ (solid curve), $k_d=0.5$ (dashed curve), and $k_d=0.7$ (dotted curve). Figure 1 agrees with the main feature of the dust-acoustic waves in a dusty plasma with the dust grain charge fluctuation: The dust-acoustic mode is damped due to the dust grain charge fluctuation.[11–14] However, Figures 2 and 3 clearly exhibit that the equilibrium dust grain charge inhomogeneity introduces a completely new extremely low-frequency stable mode and causes an unstable branch (corresponding to the real negative root) of the dust-acoustic mode.

To examine the effects of correlations in the dust fluid, we have also numerically analyzed our dispersion relation (20) for different values of the transport coefficients, such as, μ_d, η_l, and τ_m. It can be shown that for parameters used before and for $Z_{d0}=10^2$, we have $\Gamma=1$. If, we replace Z_{d0} by 3×10^2 and 10^3, but use the other parameters as before, we find that Γ becomes 10 and 160, repectively. It should be mentioned that τ_m and μ_d are already expressed as a function of Γ by (9) and (10), and typical values[18] of η_l are $1.04a_d^2\omega_{pd}$ for $\Gamma=1$, $0.08a_d^2\omega_{pd}$ for $\Gamma=10$, and $0.3a_d^2\omega_{pd}$ for $\Gamma=160$. We have shown how dispersion properties of the existing dust-acoustic mode and our new mode are modified by the presence of strong correlations in the dust fluid. The results are shown in Figs. 4–6. Figure 4 shows how the dispersion curves (ω_r vs k) of the dust-acoustic mode (upper plot) and our new mode (lower plot) are modified by strong correlations in the dust fluid (in both cases, the solid curves represent dispersion properties for $\Gamma=1$ and the dashed curves for $\Gamma=10$). Figure 5 shows how damping rate (shown in the upper plot of Fig. 3) of the dust-acoustic mode is modified by strong correlations in the dust fluid: The upper plot is for $\Gamma=1$ and the lower plot for $\Gamma=10$). Figure 6 shows how the growth rate (shown in the lower plot of Fig. 3) of the dust-acoustic mode is modified by the strong correlations in the dust fluid: The upper plot is for $\Gamma=1$ and the lower one for $\Gamma=10$). It is obvious from Figs. 4–6 that the dispersion properties of both the dust-acoustic mode and our new mode have been modified by strong correlations in the dust fluid significantly, and this modification changes irregularly with the wavelength of the mode considered.

IV. DISCUSSION

We have investigated low-frequency electrostatic dust-modes in a strongly coupled dusty plasma, accounting for the dust grain charge fluctuation and the equilibrium dust grain charge inhomogeneity. We have considered a three component unmagnetized dusty plasma comprising Boltzmann electrons and ions, and strongly coupled dust fluid where the dust grain charge at equilibrium is not constant, but varies with x. We first derive the general dispersion realtion for the low-frequency electrostatic dust-mode and then numerically analyze different roots of this general dispersion relation. The results, which have been found in this investigation, may be summarized as follows:

Downloaded 14 Feb 2001 to 130.239.166.30. Redistribution subject to AIP copyright, see http://ojps.aip.org/pop/popcpyrts.html

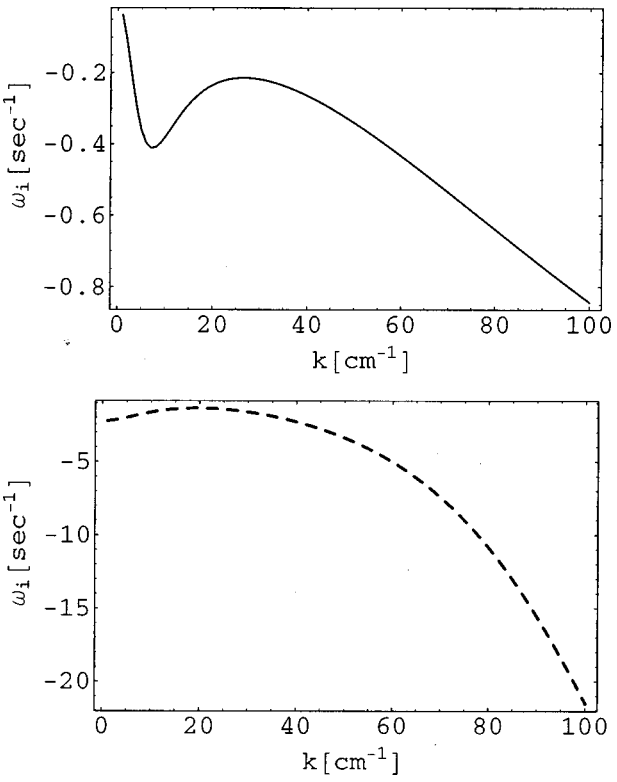

FIG. 5. The damping rate (shown in the upper plot of Fig. 3) of the dust-acoustic mode is modified by strong correlations in the dust fluid: The upper plot is for $\Gamma=1$, while the lower plot is for $\Gamma=10$.

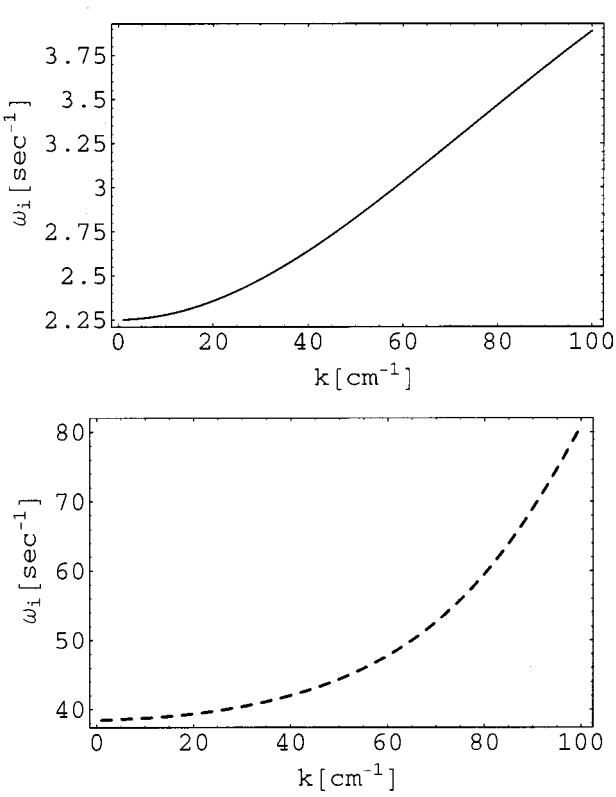

FIG. 6. The growth rate (shown in the lower plot of Fig. 3) of the dust-acoustic mode is modified by strong correlations in the dust fluid: The upper plot is for $\Gamma=1$, while the lower plot is for $\Gamma=10$.

(i) The plasma system under consideration supports a completely new stable low-frequency mode, which is only due to the equilibrium dust grain charge inhomogeneity (cf. lower plot of Fig. 2). If we neglect the effect of this dust grain charge inhomogeneity, this mode disappears and our work (without the effects of strong correlations in the dust fluid) completely agrees with a number of earlier published works[12–14] (cf. Fig. 1).

(ii) The nature of the existing dust-acoustic mode and its damping (due to the dust grain charge fluctuation) are unaffected by the dust grain charge inhomogeneity (cf. upper plots of Figs. 2 and 3).

(iii) The other branch (corresponding to the real negative root) of the dust-acoustic mode has been found to be unstable due to the effect of the equilibrium dust grain charge inhomogeneity. Also the increase in the dust grain charge inhomgeneity (i.e., k_d) stimulates the unstable wave mode for lower values of k, i.e., the wave mode of larger wavelength becomes unstable (cf. lower plot of Fig. 3).

(iv) The effects of strong correlations in the dust fluid significantly modify the dispersion properties of this new mode as well as of the dust-acoustic mode. This modification irregularly changes with the wavelength of the mode considered (cf. Fig. 4).

(v) The nature of damping of the dust-acoustic mode (which is due to the dust grain charge fluctuation) and of the growth rate of the instability of the dust-acoustic mode (which is due to the inhomogeneity in the equilibrium dust grain charge) are also found to be modified significantly by strong correlations in the dust fluid (cf. Figs. 5 and 6).

It may be stressed here that the present investigation may be useful for the study of transitions from the strongly coupled to weekly coupled regimes observed by a number of experiments during the last few years,[2–4] and for understanding the mechanism of the instability and the energy gain in a strongly coupled dusty plasma observed by a recent laboratory experiment of Nunomura *et al.*[9] This investigation may also be important for understanding the low-frequency electrostatic noise or oscillations in some astro-plasma environments, such as, white dwarf matter, interior of heavy planets, etc.

ACKNOWLEDGMENTS

One of the authors (A.A. Mamun) gratefully acknowledges the financial support of the Alexander von Humboldt-Stiftung (Bonn, Germany) and the study leave granted by the authority of Jahangirnagar University (Dhaka, Bangladesh). T. Farid gratefully acknowledges the support of the Deutscher Akademischer Austauschdienst (DAAD). This research was partially supported by the International Space Science Institute at Bern (Switzerland) through its international team ''Dust Plasma Interaction in Space.''

Downloaded 14 Feb 2001 to 130.239.166.30. Redistribution subject to AIP copyright, see http://ojps.aip.org/pop/popcpyrts.html

[1]H. Ikezi, Phys. Fluids **29**, 1764 (1986).
[2]Y. Hayashi and K. Tachibana, Jpn. J. Appl. Phys. **33**, 804 (1994).
[3]J. H. Chu and I. Lin, Phys. Rev. Lett. **72**, 4009 (1994).
[4]H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher, and D. Mohlmann, Phys. Rev. Lett. **73**, 652 (1994).
[5]M. Rosenberg and G. Kalman, Phys. Rev. E **56**, 7166 (1997).
[6]H. Totsuji, T. Kishimoto, and C. Totsuji, Phys. Rev. Lett. **78**, 3113 (1997).
[7]S. Hamaguchi, R. T. Farouki, and D. H. E. Dubin, Phys. Rev. E **56**, 4671 (1997).
[8]H. C. Lee and D. Y. Chen, Phys. Rev. E **56**, 4596 (1997).
[9]S. Nunomura, T. Misawa, N. Ohno, and S. Takamura, Phys. Rev. Lett. **83**, 1970 (1999).
[10]C. K. Goertz, Rev. Geophys. **27**, 271 (1989).
[11]N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[12]R. K. Varma, P. K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).
[13]F. Melandsø, T. K. Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 321 (1993).
[14]N. N. Rao and P. K. Shukla, Planet. Space Sci. **42**, 221 (1994).
[15]J. X. Ma and P. K. Shukla, Phys. Plasmas **1**, 1506 (1995).
[16]P. K. Shukla, in *Physics of Dusty Plasmas*, edited by P. K. Shukla, D. A. Mendis, and V. W. Chow (World Scientific, Singapore, 1996), pp. 107–121.
[17]S. Ichimaru and S. Tanaka, Phys. Rev. Lett. **56**, 2815 (1986).
[18]S. Ichimaru, H. Iyetomi, and S. Tanaka, Phys. Rep. **149**, 91 (1987).
[19]M. A. Berkovsky, Phys. Lett. A **166**, 365 (1992).
[20]P. K. Kaw and A. Sen, Phys. Plasmas **5**, 3552 (1998).
[21]J. P. Boon and S. Yip, *Molecular Hydrodynamics* (McGraw-Hill, New York, 1980).
[22]R. Abe, Prog. Theor. Phys. **21**, 475 (1959).
[23]W. L. Slattery, G. D. Doolen, and H. E. DeWitt, Phys. Rev. A **21**, 2087 (1980).

Downloaded 14 Feb 2001 to 130.239.166.30. Redistribution subject to AIP copyright, see http://ojps.aip.org/pop/popcpyrts.html