Obliquely propagating electron-acoustic solitary waves

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A theoretical investigation is carried out for understanding the properties of obliquely propagating electron-acoustic solitary waves (EASWs) in a magnetized plasma whose constituents are a cold magnetized electron fluid, hot electrons obeying a vortex-like distribution, and stationary ions. It is found that the present plasma model supports EASWs having a positive potential, which corresponds to a dip (hump) in the cold (hot) electron number density. The effects of the external magnetic field and the obliqueness are found to significantly change the basic properties (viz. the amplitude and the width) of the EASWs. The present investigation can be of relevance to the electrostatic solitary structures observed in various space plasma environments (viz. the cusp of the terrestrial magnetosphere, the geomagnetic tail, the auroral regions, etc.). © 2002 American Institute of Physics. [DOI: 10.1063/1.1462635]

An electron-acoustic (EA) wave can exist in a two-temperature (cold and hot) electron plasma. It is basically an acoustic (electrostatic) wave in which the inertia is provided by the cold electrons and the restoring force comes from the pressure of the hot electrons. The ions play the role of a neutralizing background, i.e., the ion dynamics does not influence the EA waves because the EA wave frequency is much larger than the ion plasma frequency. The spectrum of the linear EA waves, unlike that of the well-known Langmuir waves, extends only up to the cold electron plasma frequency \( \omega_{pe} = (4 \pi n_e e^2/m_e)^{1/2} \), where \( n_e \) is the unperturbed cold electron number density, \( e \) is the magnitude of the electron charge, and \( m_e \) is the mass of an electron. This upper wave frequency limit (\( \omega = \omega_{pe} \)) corresponds to a short-wavelength EA wave and depends on the unperturbed cold electron number density \( n_{e0} \). On the other hand, the dispersion relation of the linear EA waves in the long-wavelength limit (in comparison with the hot electron Debye radius \( \lambda_{De} = (k_B T_h/4 \pi n_{h0} e^2)^{1/2} \), where \( T_h \) is the hot electron temperature, \( k_B \) is the Boltzmann constant, and \( n_{h0} \) is the unperturbed hot electron number density) is \( \omega = k C_e \), where \( k \) is the wave number and \( C_e = (n_{e0} k_B T_h/n_{h0} m_e)^{1/2} \) is the electron-acoustic speed. The long-wavelength EA mode exhibits an ion-acoustic like behavior in the sense that the cold (hot) electrons play the role of the ions (electrons) in an ion-acoustic mode. The EA wave phase speed \( C_e \) must be intermediate between the cold and hot electron thermal speeds such that the wave avoids damping by both the cold and the hot electron species.

After the discovery of the existence of the EA waves, the conditions for the EA wave propagation as well as its linear properties have been investigated by many authors. The nonlinear propagation of the EA waves (as EA solitary waves) in an unmagnetized plasma has also been considered by several authors. Dubouloz et al. introduced a one-dimensional, unmagnetized, collisionless plasma composed of cold and hot electrons with motionless ions to study EA solitary waves (EASWs). Mace et al. investigated the EASWs in an unmagnetized plasma model in which the ions and the cold electron fluid are of finite temperature. They showed the existence of negative potential solitary structures associated with a compression of the cold electron density. On the other hand, Berthomier et al. considered another unmagnetized plasma model composed of an electron beam component, in addition to the cold and hot electron components, to study EA solitary waves, and showed the existence of positive potential solitary structures associated with a rarefaction of the cold electron density. However, these investigations are not valid for obliquely propagating EASWs in a magnetized plasma. The nonlinear propagation of the EA waves in a magnetized plasma has been considered by Dubouloz et al., who reported that the electric field spectrum produced by an EASW is not significantly modified by the presence of the magnetic field. Recently, Mace and Hellberg studied the properties of obliquely propagating EASWs in a magnetized plasma, and showed the existence of negative potential EASWs corresponding to a compression of the cold electron density.

To study the properties of EASWs, Mace and Hellberg considered magnetized electron and ion fluids and Maxwellian hot electrons. However, in practice the hot electrons will not have a Maxwellian distribution due to the formation of phase space holes caused by the trapping of hot electrons in the EA wave potential. Thus, in most space plasmas the hot electrons follow a vortex-like distribution. Electron trapping is common not only in space plasmas, but also in laboratory experiments, viz. in diffraction limited laser-plasma
interaction experiments where a low-velocity wave is observed. It corresponds to stimulated scattering from an EA wave and implies strong electron trapping.

In this Brief Communication, we consider a magnetized plasma model consisting of a cold, magnetized electron fluid, hot electrons obeying a nonisothermal (vortex-like) distribution, and stationary ions, and investigate the properties of small but finite amplitude obliquely propagating EASWs. We find that our plasma model supports positive potential EASWs corresponding to cold electron density holes/cavities.

The dynamics of the nonlinear EA waves (with phase speed much larger than the cold electron thermal speed but much smaller than the hot electron thermal speed) in the presence of an external magnetic field \( B_0 = \hat{z} B_0 \) is governed by

\[
\frac{\partial \mathbf{u}_c}{\partial t} + (\mathbf{u}_c \cdot \nabla) \mathbf{u}_c = \mathbf{v}_c - \omega_{ce} (\mathbf{u}_c \times \mathbf{B}).
\]

To derive a nonlinear dynamical equation for the electron-acoustic waves from (1)–(3) and (6), we must find an appropriate coordinate frame where the wave can be described smoothly. To find such a frame, we need to know the width \( \Delta \) and nonlinear speed \( v_0 \) of the wave which can be taken from an equilibrium theory using the vortex-like hot electron distributions. Thus, we find that our plasma model supports positive potential EASWs corresponding to cold electron density holes/cavities.

To model the hot electron distribution in the presence of trapped particles, we consider the hot electrons as flowing along the lines of force of the external magnetic field and employ the vortex-like electron distribution of Schamel, which satisfies the electron Vlasov equation. Thus we have

\[
f_h = f_{ht} + f_{hr},
\]

where \( f_{ht} = f_{ht,1} \) and \( f_{hr} = f_{hr,1} \). For \( \phi_c = 1 + \epsilon \), for \( 0 < \epsilon \ll 1 \), we have

\[
f_{ht,1} = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} v^2 \right) \quad \text{for} \quad |v| > \sqrt{2}\phi_c,
\]

\[
f_{hr,1} = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} \beta (v^2 - 2\phi_c) \right) \quad \text{for} \quad |v| \leq \sqrt{2}\phi_c.
\]

Here \( v \) is the hot electron speed normalized by the hot electron thermal speed \( v_{0T_h} = (k_B T_h/m)^{1/2} \) and \( \beta \) is a parameter determining the number of trapped electrons. The magnitude of \( \beta \) is defined as the ratio of the free hot electron temperature \( T_h \) to the hot trapped electron temperature \( T_{ht} \), i.e., \( |\beta| = T_h/T_{ht} \). It has been assumed that the speed of the nonlinear EA waves is much smaller than the hot electron thermal speed. We note that \( 0 < \beta < 1 \) (\( \beta = 0 \)) represents a Maxwellian (flattopped) distribution, whereas \( \beta > 0 \) represents a vortex-like excavated trapped electron distribution corresponding to an underpopulation of trapped electrons. Integrating the electron distributions over the velocity space, the hot electron number density \( n_h \) for \( \beta < 0 \) can be expressed as

\[
n_h = I(\phi) + \frac{2}{\sqrt{\pi |\beta|}} W_D(\sqrt{-|\beta|} \phi),
\]

where \( I(x) = [1 - \text{erf}(\sqrt{x})] \exp(x) \) and \( W_D(x) = \exp(-x^2) f_0 \exp(y^2) dy \) for \( \phi < 1 \) we can express (5) as

\[
n_h \approx 1 + \phi - \frac{4(1 - \beta)}{3\sqrt{\pi}} \phi^{3/2} + \frac{1}{2} \phi^2.
\]

To derive a nonlinear dynamical equation for the electron-acoustic waves from (1)–(3) and (6), we must find an appropriate coordinate frame where the wave can be described smoothly. To find such a frame, we need to know the width \( \Delta \) and nonlinear speed \( v_0 \) of the wave which can be taken from an equilibrium theory using the vortex-like hot electron distributions. Thus, we find that our plasma model supports positive potential EASWs corresponding to cold electron density holes/cavities.

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To next higher order in \( \epsilon \), from the continuity equation, the \( z \)-component of the momentum equation, and Poisson’s equation, we obtain

\[
\frac{\partial n_c^{(1)}}{\partial \tau} - v_0 \frac{\partial n_c^{(2)}}{\partial \xi} + l_z \frac{\partial u_c^{(2)}}{\partial \xi} + l_z \frac{\partial u_c^{(2)}}{\partial \xi} + l_z \frac{\partial u_c^{(2)}}{\partial \xi} = 0, \tag{10a}
\]

\[
\frac{\partial n_c^{(1)}}{\partial \tau} - v_0 \frac{\partial n_c^{(2)}}{\partial \xi} - \alpha l_z \frac{\partial \phi^{(2)}}{\partial \xi} = 0, \tag{10b}
\]

\[
\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} - \frac{1}{\alpha} \left( \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{4(1-\beta)}{3\sqrt{\pi}} [\phi^{(1)}]^{3/2} \right) = 0. \tag{10c}
\]

Combining (9) and (10) we deduce the modified Korteweg-de Vries (KdV) equation

\[
\frac{\partial \phi^{(1)}}{\partial \tau} + \frac{l_z (1-\beta)}{\sqrt{\pi}} \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{l_z^2}{2} \left[ 1 + \frac{l_z}{\omega_c^2} \right] \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0. \tag{11}
\]

It is important to note here that the modified KdV equation (11) is only valid for a vortex-like excited trapped hot electron distribution (i.e., for \( \beta < 1 \)). This equation is not valid for a Maxwellian hot electron distribution (\( \beta = 1 \)) because the nonlinear effect caused by the term \( \phi^2 \) (which is usually small in comparison with that caused by the term \( \phi^{3/2} \) for \( \beta < 0 \)) has been dropped out due the ordering we have used. However, in the case a Maxwellian hot electron distribution (\( \beta = 1 \)) (considered by Mace and Hellberg\(^9\)) the quantities \( \Delta \) and \( v_0 \) behave as \( \Delta \sim \epsilon^{-1/2} \) and \( (v_0-1) \sim \epsilon \), respectively, the ordering that we have considered here is no longer applicable, and we must return to the ordering of Washimi and Taniuti\(^{14}\) yielding the KdV equation of standard form.\(^{14}\)

A stationary solitary wave solution of the modified KdV equation (11) can be obtained by transforming the space variable to \( \xi = \xi - u_0 \tau \) and by imposing the boundary conditions for localized perturbations, viz. \( \phi^{(1)} \to 0 \), \( d\phi^{(1)}/d\xi \to 0 \), \( d^2\phi^{(1)}/d\xi^2 \to 0 \) at \( \xi \to \pm \infty \). Thus, the steady state solution of (11) can be expressed as

\[
\phi^{(1)} = \Psi \text{sech}^2[(\xi - u_0 \tau)/\Delta], \tag{12}
\]

where \( \Psi = [15u_0 \sqrt{\pi}/8\sqrt{1-\beta}]^2 \) and \( \Delta = \sqrt{8l_z(1+(1-l_z^2)/\omega_c^2)} \) are, respectively, the amplitude and the width of the solitary waves. Equation (12) reveals the existence of small but finite amplitude EASWs with a positive potential. This corresponds to a dip (hump) in the cold (hot) electron number density, since \( n_c^{(1)} = -\alpha \phi^{(1)} \). We find that the amplitude \( \Psi \) is independent of the magnitude of the external magnetic field, but is inversely proportional to \( l_z^2 \) and \( (1-\beta)^2 \). The variation of the amplitude \( \Psi \) with \( \theta = \cos^{-1} l_z \) for different values of \( \beta \) is also shown in Fig. 1. We also find that for \( (1-l_z^2) \approx \omega_c^2 \) the width \( \Delta \) is independent of \( \omega_c \), but directly proportional to \( l_z^2 \). However, for \( \omega_c \ll 1 \) the width \( \Delta \) strongly depends on \( \omega_c \) and \( l_z \), as depicted in Fig. 2, which shows the variation of \( \Delta \) with \( \theta \) for different values of \( \omega_c \).

If we compare our results with that of Mace and Hellberg,\(^9\) who have considered Maxwellian hot electron distribution, we find that (i) the polarity of the solitary structure for a vortex like hot electron distribution is opposite to that for a Maxwellian hot electron distribution, (ii) the Maxwellian hot electron distribution provides a KdV equation of the standard form,\(^{14}\) whereas the vortex-like hot electron distribution gives rise to a modified KdV equation [cf. (11)] which exhibits a stronger nonlinearity, corresponding to smaller width and higher velocity of the wave, (iii) in both cases the variation of the width with \( \omega_c \) and \( l_z \) is the same, and (iv) for a vortex-like hot electron distribution the amplitude is inversely proportional to \( l_z^2 \) and \( (1-\beta)^2 \), whereas for a Maxwellian hot electron distribution it is inversely proportional to \( l_z \) and \( (1+3\alpha) \).

To compare our result (the amplitude of the electrostatic potential) with that observed in the auroral zone, we choose a set of available parameters corresponding to the dayside auroral zone where an electric field amplitude \( E_0 \approx 100 \) mV/m has been observed:\(^8,17\) \( T_e = 5 \) eV, \( T_i = 250 \) eV, \( n_{e0} = 0.5 \) cm\(^{-3} \), \( n_{i0} = 2.5 \) cm\(^{-3} \). These parameters correspond to \( \lambda_{De} \approx 7430 \) cm and the normalized electrostatic wave potential amplitude \( \Psi = E_0 \lambda_{De}/(e k_B T_e) \approx 0.03 \) which is obtained for \( \alpha = n_{i0}/n_{e0} = 5 \), \( \omega_c = 5 \), \( u_0 = 1.2 \), \( \beta = -0.5 \) and \( \theta = 58^\circ \) (cf. the solid curve of Fig. 1), \( \beta = -0.7 \) and \( \theta = 61^\circ \) (cf. dotted curve of Fig. 1), and \( \beta = -0.9 \) and \( \theta = 63^\circ \) (cf. dashed curve of Fig. 1). This means that our results completely agree with the observations from the Viking satellite in the dayside auroral zone where an electric field amplitude of \( \approx 100 \) mV/m is observed.\(^8,17\) In conclusion, we stress that our investigation should be useful in understand-
ing salient features of localized electrostatic perturbations in space and laboratory plasmas.

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