New resonance and cut-off for low-frequency electromagnetic waves in dusty magnetoplasmas

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The dispersion properties of low-frequency (in comparison with the ion gyrofrequency) dust-electromagnetic waves in a four-component magnetized dusty plasma have been investigated. It has been shown that the simultaneous presence of positive and negative inertial dust grains introduces new resonance and cut-off for dust-electromagnetic waves, and a sharp forbidden-frequency-band in between the two branches of the propagating dust-electromagnetic waves. The two wave branches have been found to have frequencies of the order of negative and positive dust gyrofrequencies, respectively. The effects of space and laboratory dusty plasma parameters on these newly found resonance and cut-off and on the dispersion properties of the dust-electromagnetic wave branches have also been examined. © 2004 American Institute of Physics.

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Charged dust particles are ubiquitous in space and laboratory plasmas, and are found to play a crucial role in supporting new electrostatic\(^1,2\) and electromagnetic\(^3,4\) modes in a uniform dusty plasma without and with an external magnetic field. In a magnetized dusty plasma with negatively charged dust grains, Shukla\(^5\) and Rao\(^6\) predicted new cut-off frequencies for low-frequency (in comparison with the ion gyrofrequency) magnetic field aligned circularly polarized and transversely propagating (with respect to the external magnetic field direction) elliptically polarized compressional magnetoacoustic waves. The underlying physics and the details of numerous wave modes in dusty plasmas are contained in two textbooks.\(^5,6\)

The presence of positively charged dust particles have also been observed in different regions of space, viz., cometary tails,\(^7–11\) Jupiter’s magnetosphere,\(^12\), etc. There are three principal mechanisms by which a dust grain becomes positively charged.\(^13\) These are (i) photoemission in the presence of a flux of ultraviolet (UV) photons, (ii) thermionic emission induced by radiative heating, and (iii) secondary emission of electrons from the surface of the dust grains.

On the other hand, Chow et al.\(^7\) have theoretically shown that due to the size effect on secondary emission insulating dust grains with different sizes in space plasmas can have the opposite polarity (smaller ones being positive and larger ones being negative). This is mainly due to the fact that the excited secondary electrons have shorter (longer) distances to travel reach the surface of the smaller (larger) dust grains (for details we refer to Chow et al.\(^7\)). Chow et al.\(^7\) have found that insulating grains with diameters 0.01 \(\mu\)m and 1 \(\mu\)m have opposite polarity (with smaller ones being positive). These values may be in the range of the inferred values in different regions of planetary ring systems, comets, interplanetary medium, supernova remnants, etc.\(^8,9\)

There is also direct evidence for the existence of both positively and negatively charged dust particles in cometary tails.\(^8–11\) Motivated by ample theoretical\(^7\) and experimental\(^8–13\) grounds for the existence of positively charged dust particles or dust particles of opposite polarity (smaller ones being positive and larger ones being negative), Mamun and Shukla\(^14\) considered an unmagnetized dusty plasma containing positively and negatively charged dust particles by assuming a complete depletion of background electrons and ions, and theoretically investigated the properties of longitudinal electrostatic waves.

However, dusty plasmas in space and in some laboratory experiments are embedded in an external magnetic field. Accordingly, in this Brief Communication we present an investigation of the dispersion properties of low-frequency compressional dust-electromagnetic waves in a low-\(\beta\) (= plasma particle kinetic energy density/magnetic field energy density) multicomponent dusty magnetoplasma composed of electrons, ions, and negatively as well as positively charged dust particles. We account for the dynamics of negative (larger) and positive (smaller) dust particles, which are pushed by the magnetic field pressure. It is found that the simultaneous presence of negatively and positively charged inertial dust grains introduce new resonance and cut-off for the low-frequency dust electromagnetic waves, and a sharp forbidden frequency band (FFB) in between these two low-frequency dust-electromagnetic wave branches.

In our multicomponent dusty magnetoplasma, we have at equilibrium \(n_{e0} = n_{i0} = Z_1 n_{20} + Z_2 n_{10}\), where \(n_{j0}\) is the equilibrium number density of the species \(j\) \((j= e, i, 1, 2\) for electron, ion, positive dust, and negative dust, respectively) and \(Z_1 (Z_2)\) is the number of protons (electrons) residing onto a positive (negative) dust particle. We are interested in

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elliptically polarized electromagnetic waves whose frequencies are much smaller than the ion gyrofrequency, i.e., $|\delta| \ll \omega_{ci}$, where $\delta = \delta / \omega_{ci}$, $\omega_{ci} = eB_0/m_i c$, $e$ is the proton charge, $B_0$ is the magnitude of the ambient magnetic field $B_0$, $m_i$ is the ion mass, and $c$ is the speed of light in vacuum. In the presence of such low-frequency electromagnetic waves, the dynamics of electrons, ions, and positive as well negatively charged dust particles are described by a set of closed linearized equations:

$$0 \simeq E + \frac{1}{c} V \times B_0,$$  \hspace{0.3cm} (1)

$$0 \simeq E + \frac{1}{c} V \times B_0,$$  \hspace{0.3cm} (2)

$$\partial_t V_1 = \frac{Z_1 e}{m_1} \left( E + \frac{1}{c} V \times B_0 \right),$$  \hspace{0.3cm} (3)

$$\partial_t V_2 = - \frac{Z_2 e}{m_2} \left( E + \frac{1}{c} V \times B_0 \right),$$  \hspace{0.3cm} (4)

$$\nabla \times E = - \frac{1}{c} \partial_t B,$$  \hspace{0.3cm} (5)

$$\nabla \times B = \frac{4\pi}{c} J,$$  \hspace{0.3cm} (6)

where $V_j$ is the velocity of the species $j$, $m_1$ ($m_2$) is the mass of a positively (negatively) charged dust particle, $E$ and $B$ are the perturbed electric and magnetic fields, and $J = e(n_0 V_1 - n_0 V_2 + Z_1 n_1 V_1 - Z_2 n_2 V_2)$ is the net plasma current. The cold plasma approximation used here is valid for low-frequency compressional electromagnetic waves whose phase speeds are larger than the dust sound speed in a low-$\beta$ plasma.

We now assume that $B_0$ and $B$ both point along the $z$-axis, $E$ lies in the $x$-$y$ plane, and $J$ is almost along the $y$-axis. We can, therefore, consider an approximated one-dimensional problem in which all perturbed quantities will depend only on $x$ and $t$. Therefore, using (1)–(6) we have

$$\partial_t V_{1x} = \omega_{ci} \mu_2 (V_{1y} - V_{2y}) + \frac{\mu_1 V_{A1}^2}{B_0} \partial_x B,$$  \hspace{0.3cm} (7)

$$\partial_t V_{1y} = - \omega_{ci} \mu_2 (V_{1x} - V_{2x}),$$  \hspace{0.3cm} (8)

$$\partial_t V_{2x} = - \omega_{ci} \mu_2 (V_{1y} - V_{2y}) - \frac{\mu_2 V_{A2}^2}{B_0} \partial_x B,$$  \hspace{0.3cm} (9)

$$\partial_t V_{2y} = \omega_{ci} \mu_2 (V_{1x} - V_{2x}),$$  \hspace{0.3cm} (10)

$$\partial_t V_{1z} = - \omega_{ci} \mu_2 (V_{1y} - V_{2y}) = \frac{1}{B_0} \partial_z B,$$  \hspace{0.3cm} (11)

where $\mu_1 = \mu/(1 - \mu)$, $\mu_2 = 1/(1 - \mu)$, $\mu = Z_1 n_1 / Z_2 n_2$, $\omega_{ci} = Z_1 eB_0 / m_1 c$, $\omega_{ci} = Z_2 eB_0 / m_2 c$, $V_{A1} = B_0 / \sqrt{4\pi e n_{10} m_1}$, and $V_{A2} = B_0 / \sqrt{4\pi e n_{20} m_2}$.

Substituting (8) into (7) and (10) into (9) we can easily eliminate $V_{1y}$ and $V_{2y}$. Accordingly, we have

$$(\delta_1^2 + \mu_2 \omega_{ci}) V_{1x} - \mu_2 \omega_{ci} V_{2x} = \frac{\mu_1 V_{A1}^2}{B_0} \delta_1^2 B,$$  \hspace{0.3cm} (12)

$$\mu_2 \omega_{ci} V_{1x} - (\delta_2^2 + \mu_1 \omega_{ci}) V_{2x} = \frac{\mu_2 V_{A2}^2}{B_0} \delta_2^2 B,$$  \hspace{0.3cm} (13)

where $\omega_{ci} = \mu_2 \omega_{ci} + \mu_2 \omega_{ci} - \omega_{ci} (\sigma + \mu)/(1 - \mu)$, $\sigma = Z_1 m_1 / Z_2 m_2$, and $\delta_1^2 = \delta_2^2$.

We now perform a normal mode analysis by assuming that $V_{1x}$, $V_{2x}$, and $B$ are proportional to $\exp[-i(\omega t - kx)]$, where $\omega$ is the wave frequency and $k$ is the propagation constant. We then obtain from (11)–(13) a dispersion relation,

$$\frac{\omega^2}{k^2} V_{A1}^2 \left( 1 - \frac{\omega_1^2}{\omega^2} \right),$$  \hspace{0.3cm} (14)

where $V_{A1}^2 = \mu_1 V_{A1}^2 + \mu_2 V_{A2}^2 = V_{A2}^2 (1 + \sigma \mu)/(1 - \mu)^2$ and

$$\omega^2 = \omega_{ci} (\mu_1 \mu_2 V_{A2}^2 - \omega_{ci} \mu_1 V_{A1}^2)/V_{A1}^2$$  \hspace{0.3cm} (15)

It is obvious that if we neglect any one of the two dust components, $\omega_{ci}$ and $\omega_{ci}$ become zero and (14) reduces to a dispersion relation for the fast Alfvén waves associated with the other dust component. That is, for example, if we neglect the contribution of positive dust particles (i.e., $\mu = 0$ and $\sigma = 0$), we have the fast Alfvén waves $\omega^2 = V_{A1}^2 = B_0^2 (4\pi e n_{10} m_1)$ in which the magnetic pressure $B_0^2/4\pi$ gives rise to the restoring force, and the negative dust mass density $n_{20} m_2$ provides the inertia.

We now examine the contribution of both the positive and negative dust particles, which involve two new terms denoted by $\omega_{ci}$ and $\omega_{ci}$, respectively. Equation (14) indicates that the combined effects of these two dust components introduce a (new) resonance $(k = \infty)$ at

$$\omega = \omega_{ci} = \sqrt{\frac{\sigma (\sigma + \mu)}{1 + \sigma \mu}},$$  \hspace{0.3cm} (16)

a (new) cut-off $(k = 0)$ at

$$\omega = \omega_{ci} = \frac{\sigma + \mu}{1 - \mu},$$  \hspace{0.3cm} (17)

and a forbidden frequency band (FFB) between these two frequencies, $\omega_{ci} - \omega_{ci}$.

We have numerically shown how $\omega_{ci}$, $\omega_{ci}$, and FFB vary with $\mu$ and $\sigma$. The numerical results are displayed in Fig. 1. The upper plot of Fig. 1 shows that $\omega_{ci} = \omega_{ci}$ at $\mu = 0.455$, and $\omega_{ci} = \omega_{ci}$. For $\mu < (>) \omega_{ci}$, $\omega_{ci}$ increases as we decrease or increase $\mu$ from its critical value 0.455. How the width of the FFB changes with $\mu$ is clearly indicated by the dashed regions in the upper plot of Fig. 1. The lower plot of Fig. 1 depicts how the width of the FFB (indicated by the dashed lines) changes with $\sigma$. It clearly indicates that as we increase $\sigma$, the width of the FFB increases.
(14), and have shown the dispersion properties of the low-frequency dust-electromagnetic waves with the cut-off, resonance, and FBB for \( \sigma = 10 \) and \( \mu = 0.1 \) (upper plot) and \( \mu = 0.8 \) (lower plot). The upper plot of Fig. 2 shows that for \( \sigma = 10 \) and \( \mu = 0.1 \) the cut-off frequency, \( \omega_c = 3.45\omega_{z2} \), is higher than the resonance frequency \( \omega_r = 7.35\omega_{z2} \). This means that the FBB is \( \omega_r - \omega_c = 3.9\omega_{z2} \), and the frequency range of the mode is \( 0 < \omega < 3.45\omega_{z2} \) before the FBB and \( 7.35\omega_{z2} < \omega \leq \omega_c \) after the FBB. The variation of \( k \) with \( \omega \) before the FBB and after the FBB are indicated by the dashed curve and solid curve, respectively. The lower plot of Fig. 2 shows that for \( \sigma = 10 \) and \( \mu = 0.8 \) the resonance frequency, \( \omega_r = 7.25\omega_{z2} \), is higher than the cut-off frequency \( \omega_c = 3.5\omega_{z2} \). This indicates that the FBB is \( \omega_r - \omega_c = 3.9\omega_{z2} \), and the frequency range of the mode is \( 0 < \omega < 3.5\omega_{z2} \) before the FBB and \( 7.25\omega_{z2} < \omega \leq \omega_c \) after the FBB. The variation of \( k \) with \( \omega \) before the FBB and after the FBB are indicated by the solid curve and dashed curve, respectively.

We now apply our theoretical results to space and laboratory dusty plasmas. For space dusty plasmas, particularly for cometary dust plasma parameters, we typically have \( B_0 = 10^{-2} \text{ G}, m_1 = 10^{-17} \text{ g}, m_2 = 10^{-15} \text{ g}, Z_1 = 10^2, Z_2 = 10^3, n_1 = 10^{-3} \text{ cm}^3, \) and \( n_2 = 10^{-3} \text{ cm}^3 \). Thus, with \( \mu = 0.1, \sigma = 10, \) and \( \omega_{z2} \approx 1.6 \times 10^{-4} \text{ s}^{-1} \), we have \( \omega_r \approx 5.5 \times 10^{-4} \text{ s}^{-1}, \omega_c \approx 1.17 \times 10^{-3} \text{ s}^{-1}, \) and FBB \( \approx 6.24 \times 10^{-4} \text{ s}^{-1} \). The dispersion properties of the dust-electromagnetic waves before and after the band corresponding to these laboratory dusty plasma parameters are graphically shown in the lower plot of Fig. 2. On the other hand, for laboratory dusty plasma parameters, \( B_0 = 10^4 \text{ G}, m_1 = 5 \times 10^{-14} \text{ g}, m_2 = 5 \times 10^{-14} \text{ g}, Z_1 = 10^3, Z_2 = 10^4, n_1 = 8 \times 10^5 \text{ cm}^{-3}, \) and \( n_2 = 5 \times 10^5 \text{ cm}^{-3}, \) i.e., \( \mu = 0.8, \sigma = 10 \) and \( \omega_{z2} \approx 32 \text{ s}^{-1} \), we have \( \omega_r \approx 10^7 \text{ s}^{-1}, \omega_c \approx 232 \text{ s}^{-1}, \) and FBB \( \approx 200 \text{ s}^{-1} \). The dispersion properties of the dust-electromagnetic waves before and after the band corresponding to these laboratory dusty plasma parameters are graphically shown in the lower plot of Fig. 2. It is important to note that when both dust-components are of same polarity (viz., negatively charged, i.e., \( Z_1 \rightarrow -Z_1 \)), the resonance frequency \( \omega_r \) remain unchanged, but the cut-off frequency \( \omega_c \) is reduced by a factor \((1 - \mu)/(1 + \mu)\). The consideration of similar polarity dust grains \( (Z_1 \rightarrow -Z_1) \) has no effect on the nature of the dispersion curves before and after the forbidden frequency band; it only increases the wave propagation constant \( (k) \) by a factor \((1 + \mu)/(1 - \mu)\).

To summarize, we have presented the linear propagation characteristics of low-frequency dust-electromagnetic waves in low-temperature multicomponent dusty magnetoplasmas.
whose constituents are electrons, ions, and charged dust grains of opposite polarity. Since we have focused on waves with frequency much smaller than the ion gyrofrequency, both electrons and ions suffer the $c\mathbf{E} \times \mathbf{B}_0 / B_0^2$ drift, while the dynamics of charged dust particles is controlled by the electric and Lorentz forces. It is found that the simultaneous presence of negative and positive dust components introduce new resonance and cut-off for the low-frequency dust electromagnetic branches, and a sharp forbidden frequency band in between the two branches of the low-frequency dust-electromagnetic modes. The latter have been found to have frequencies of the order of negative and positive dust gyro-frequencies, respectively. We have also examined the effects of different dusty plasma parameters (viz. $\mu$ and $\sigma$ corresponding to space and laboratory dusty plasma parameters) on these newly found resonance and cut-off (determining the width of the FFB) and on the dispersion properties of the low-frequency dust-electromagnetic wave branches. We propose to conduct dusty plasma laboratory experiments in an external magnetic field to verify our theoretical prediction of new resonance and cut-off for the low-frequency dust electromagnetic waves.

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