Oscillonic Mach cones in a dusty magnetoplasma

P. K. Shukla, A. A. Mamun, and B. Eliasson
Institut für Theoretische Physik IV, Ruhr-Universität Bochum, D-44780 Bochum, Germany

G. E. Morfill
Max Planck Institut für Extraterrestrische Physik, 85740 Garching, Germany

(Received 18 March 2004; accepted 5 May 2004; published online 20 July 2004)

The possibility for the formation of Mach cones in a dusty magnetoplasma containing elongated charged dust grains has been examined theoretically. It is predicted that the Mach cones can be formed by dipole oscillons (wave modes associated with the oscillations of dust dipoles around their mean positions) in a laboratory plasma containing elongated (rodlike) negatively charged dust grains. An experiment in radio-frequency dusty plasma discharges should be conducted for verifying the theoretical prediction that has been made here.

© 2004 American Institute of Physics. [DOI: 10.1063/1.1766303]

Dusty plasmas containing elongated dust grains have received a great deal of interest not only because of their ubiquity in space and laboratory plasmas, but also because of their vital role in modifying waves and instabilities in dusty plasmas. Oscillons in which the restoring force comes from the thermal pressures of electrons and ions, while the inertia is provided by the moment of inertia of dust dipoles that oscillate around their equilibrium positions. The dispersion properties of this new wave mode and associated instabilities in unmagnetized or magnetized plasmas containing electrons, ions, and elongated (rodlike) charged dust grains have been explicitly investigated by many authors.

It has been shown that the dipoles of charged elongated rodlike dust grains suspended in a plasma oscillate and introduce therein a new wave mode known as the dipole oscillon in which the restoring force comes from the thermal pressures of electrons and ions, while the inertia is provided by the moment of inertia of dust dipoles that oscillate around their equilibrium positions. The dispersion properties of this new wave mode and associated instabilities in unmagnetized or magnetized plasmas containing electrons, ions, and elongated (rodlike) charged dust grains have been explicitly investigated by many authors.

A rapid growth of dusty plasma research has stimulated a renewed interest in many classical problems which are significantly modified by new physical effects associated with charged dust grains of different shapes and sizes. One of these classical problems is the formation of Mach cones. Mach cones are formed by an object moving with supersonic speed in a medium. The moving object creates expanding waves which are spherical in three dimensions (and circular in two dimensions), and the V-shaped cone is formed by the superposition of the expanding waves. Recently, the possibility for the formation of Mach cones in space and laboratory dusty plasmas has attracted considerable attention because Mach cones can play a vital role as potential diagnosis methods, since they can be directly viewed from the outside of the dusty plasma system, and can be used for deducing information about the physical state of the ambient dusty plasma. The possibility for the formation of dust-acoustic and dust-magnetosonic Mach cones in Saturn’s dusty rings has been rigorously investigated. However, in a laboratory dusty plasma containing elongated rodlike charged dust grains, ions, and electrons, a question may arise: Can Mach cones be formed by dust dipole oscillons in such a laboratory dusty plasma? Our present theoretical work is aimed to provide an answer to this question. It is found here that the Mach cones can be formed by dipole oscillons in a laboratory plasma containing elongated dust grains.

We consider the propagation of the low-frequency wave in a dusty magnetoplasma. The ion gyroradius, the ion thermal speed, and the ion temperature, we cannot use the fluid theory to determine the ion density perturbation n_{i1} in the electrostatic wave potential. Then, we have to resort to the ion Vlasov equation to calculate the ion density perturbation. For arbitrary values of b_i = k_i^2 \rho_i^2, where k_i is the perpendicular (to the external magnetic field) component of the propagation vector \mathbf{k}, we have n_{i1} = -(k^2/4\pi) \chi_i \phi, where the ion susceptibility is

\chi_i = \frac{1}{k^2} \frac{\Lambda_i}{\Lambda_i(b_i)} \left[ 1 - \frac{\Lambda_i(b_i)}{\Lambda_i(b_i)} \frac{\omega^2}{\omega_c^2} \right]. (1)
Here $\lambda_D = (T_e/4\pi n_{i0}e^2)^{1/2}$ is the ion Debye radius, $\Lambda_{0,i} = I_{0,i}\exp(-b_i)$, and $I_0 (I_1)$ is the zero- (first-) order modified Bessel function.

The electron density perturbation, for $b_e = k_i^2 v_{te}^2 / \omega_{ce}$, $\ll 1$ and $\omega \ll \omega_{ce}$, is given by $n_e = (k_i^2 / 4\pi e^2) \chi_e \phi$, where the electron susceptibility is

$$\chi_e = \frac{\omega_{pe}^2 k_i^2}{\omega_{ce}^2 k_i^2} - \frac{\omega_{pe}^2}{\omega^2 - \gamma_e k_i^2 v_{te}^2 k_i^2^2}.$$  \hspace{1cm} (2)

Here $v_{te} = (T_e / m_e)^{1/2}$ is the electron thermal speed, $\omega_{ce} = eB_0 / m_e c$ is the electron gyrofrequency, $\omega_{pe} = (4\pi n_{i0}e^2 / m_e)^{1/2}$ is the electron plasma frequency, $\gamma_e$ is the electron adiabatic index, $k_i$ is the $z$ component of the propagation vector $k$, and $m_e$ is the electron mass. We consider $\chi_e$ in two limiting cases. First, for $\omega \gg \sqrt{\gamma_e k_i v_{te}}$ and $(k_i / k_i^2)^2 \ll (\omega / \omega_{ce})^2$, Eq. (2) reduces to

$$\chi_e = \frac{1}{\kappa^2 \lambda_{De}}.$$  \hspace{1cm} (3)

and second, for $\omega \ll k_i v_{te}$ (where $\gamma_e = 1$ is used) we have from Eq. (2)

$$\chi_e = \frac{1}{\kappa^2 \lambda_{De}}.$$  \hspace{1cm} (4)

where $\lambda_{De} = (T_e / 4\pi n_{i0}e^2)^{1/2}$ is the electron Debye radius. We note that Eq. (4) corresponds to three-dimensional ($E \times B$, the polarization and parallel to $2B_0$ drifts) motions in which electrons are rapidly thermalized along the $z$ direction.

We are interested in the electrostatic waves with the frequency much larger than the dust gyrofrequency. Thus, the dust grains can be considered unmagnetized. The dust number density perturbation can, therefore, be expressed as $n_{d1} = (k_i^2 / 4\pi A_{D1}) \chi_d \phi$, where $\chi_d$ is the dust susceptibility for $\omega_{pd} \ll \Omega_0$, and $\Omega_0 \ll \omega$, where $\omega_{pd} = (4\pi n_{i0}e^2 / m_d)^{1/2}$ is the dust plasma frequency, $\Omega_0 = (4\pi n_{i0}e^2 / m_d)^{1/2}$ is the oscillon frequency, $\Omega_0 - M / l$, is the magnitude of the dipole moment, $l$ is the moment of inertia of the dipole, and $M$ is the $z$ component of the dust angular momentum, the dust susceptibility is given by

$$\chi_d = -\frac{\Omega_0^2 k_i^2}{\omega^2 k_i^2}.$$  \hspace{1cm} (5)

Substituting Eqs. (1) and (3)–(5) in (1) $\chi_e + \chi_i + \chi_d = 0$, the dispersion relation involving $\Psi = V_p / V_d$ (a ratio of the wave phase speed $V_p = \omega / k$ to the dust particle speed $V_d$ involved in creating Mach cones) can be expressed as

$$\alpha_1 + \frac{2V_p^2 \lambda_{D1}}{\kappa^2 \omega_{ce}^2} \Psi - \frac{\Omega_0^2 k_i^2}{k_i^2 V_d V_{te}^2} = 0,$$  \hspace{1cm} (6)

where $\alpha_1 = 1 + \omega_{pd}^2 k_i^2 / \omega_{ce}^2 + (1 - \Lambda_0) / k_i^2 \lambda_{D1}^2$ corresponds to the first limiting case $\omega \gg \sqrt{\gamma_e k_i v_{te}}$, and $\alpha_2 = 1 + 1 / k_i^2 \lambda_{D1}^2 + (1 - \Lambda_0) / k_i^2 \lambda_{D1}^2$ corresponds to the second limiting case $\omega \ll k_i v_{te}$. We note that the dust particle speed $V_d$ in Eq. (6) is considered as a free parameter.

Mach cones can be formed by any perturbation waves [e.g., waves defined by Eq. (6) in our case] if the object (viz., a dust particle in our case) speed $V_d$ is larger than the wave phase speed $V_p$, i.e., $\Psi = V_p / V_d < 1$. If this condition is satisfied, the Mach cone opening angle $\theta$ is given by $\theta = \sin^{-1} \Psi$, where $\Psi$ can be obtained from Eq. (6). In laboratory dusty plasmas, one can achieve $V_d$ larger than $V_p$ by accelerating an externally launched dust particle in plasmas by the pressure of laser beams that are employed for diagnostic purposes.

We first analytically analyze Eq. (6) in some different special cases which are as follows:

a. $\omega \gg \sqrt{\gamma_e k_i v_{te}}$. We consider two special cases $b_i \ll 1$ and $b_i \gg 1$, and examine the possibility for the formation of Mach cones. For $b_i \ll 1$ we have $\omega = \Omega_0 / (1 + \omega_{pd}^2 / \omega_{ce}^2 + \omega_{pd}^2 / \omega_{ce}^2)$ which is nondispersive and cannot participate in the formation of Mach cones. On the other hand, for $b_i \gg 1$ we have

$$\omega = \frac{k_i \lambda_{D1} \Omega_0}{[1 + k_i^2 \lambda_{D1}^2 (1 + \omega_{pd}^2 / \omega_{ce}^2)]^{1/2}}.$$  \hspace{1cm} (7)

Equation (7) represents dispersive dipole oscillons, where the restoring force arises due to the thermal pressure of ions and the inertia is provided by the moment of inertia of dipole that oscillate around their equilibrium positions. Since it is a dispersive wave, it can participate in the forma-
tion of Mach cones. It implies that the dispersive oscillons [defined by Eq. (7)] form the Mach cones if \( V_d > V_p \), where \( V_p = \omega/k \) is determined from Eq. (7).

b. \( \omega \ll k, v_{te} \). We again consider two special cases \( b_i \ll 1 \) and \( b_i \gg 1 \), and examine the possibility for the formation of Mach cones. For \( b_i \ll 1 \) we have

\[ \omega = \frac{k \lambda_{D_e} \Omega_{r}}{[1 + k^2 \lambda_{D_e}^2 (1 + \omega_p^2/\omega_{ci}^2)]^{1/2}}, \tag{8} \]

which represents dispersive dipole oscillons, where the restoring force is provided by the thermal pressure of electrons and the inertia is provided by the moment of inertia of dust dipoles. The dispersive oscillons, defined by Eq. (8), form the Mach cones if \( V_d > V_p \), where \( V_p \) is obtained from Eq. (8). On the other hand, for \( b_i \gg 1 \) we have

\[ \omega = \frac{k \lambda_{D_e} \Omega_{r}}{(1 + k^2 \lambda_{D_e}^2)^{1/2}}, \tag{9} \]

where \( \lambda_D = \lambda_p \lambda_{D_e}/(\lambda_{D_e}^2 + \lambda_p^2)^{1/2} \). Equation (9) again represents dispersive dipole oscillons, where the restoring force is provided by the thermal pressures of electrons and ions, and the inertia is provided by the moment of inertia of dust dipoles. The dispersive oscillons, defined by Eq. (9), form the Mach cones if \( V_d > V_p \), where \( V_p = \omega/k \) is determined from Eq. (9).

To analyze the possibility for the formation of the Mach cones associated with the waves defined by Eq. (6) when \( k \rho_i \) is comparable to 1, we numerically analyze Eq. (6) and find \( \Psi = 1 \) curves in \( (V_d, k \rho_i) \) space for typical laboratory dusty plasma parameters: \( T_e = 10 \text{ eV}, T_i = 0.2 T_e, n_{io} = 2.5 \times 10^8 \text{ cm}^{-3}, Z_d = 5 \times 10^4, \text{ and } n_{io} = 10^3 \text{ cm}^{-3} \). The numerical results are displayed in Figs. 1–3.

Figure 1 shows the \( \Psi = 1 \) curves in \( (V_d, k \rho_i) \) space for both \( \omega \gg \sqrt{\gamma_e k, v_{te}} \) (upper plot) and \( \omega \ll k, v_{te} \) (lower plot). The \( \Psi = 1 \) curves in \( (V_d, k \rho_i) \) space will, obviously, determine the critical values of the dust particle speed \( V_d \) and the corresponding wavelength \( \lambda = 2 \pi/k \) for which the Mach cones are formed. The regions above the \( \Psi = 1 \) curves correspond to \( \Psi < 1 \), i.e., correspond to a regime for which the Mach cones in a magnetoplasma containing elongated dust grains are formed. It implies that for a wave of fixed wavelength as we increase \( \Omega_r \), we need a dust particle of higher speed in order for the formation of the Mach cones in both the limits. Figure 2 shows the variation of the Mach cone opening angle with \( k \rho_i \) and \( B_0 \) for typical laboratory dusty plasma parameters. It indicates that the Mach cone opening angle \( \theta = \sin^{-1} \Psi \) decreases with the wavelength, but increases with the magnitude of the external field. We have found that the effect of the external magnetic field is slightly stronger for \( \omega \ll k, v_{te} \) (lower plot) than that for \( \omega \gg \sqrt{\gamma_e k, v_{te}} \) (upper plot). Figure 3 depicts the variation of the
Mach opening cone angle with $k \rho_i$ and $\Omega_r$ for typical laboratory dusty plasma parameters. It indicates that the Mach cone opening angle $\theta$ decreases with the wavelength, but increases with $\Omega_r$. It is observed here that the effect of $\Omega_r$ is stronger for $\omega \ll k v_{te}$ (lower plot) than that for $\omega \gg \sqrt{\gamma_e k v_{te}}$ (upper plot).

To summarize, we have investigated the possibility for the formation of the Mach cones in a dusty magnetoplasma whose constituents are magnetized electrons and ions as well as unmagnetized negatively charged elongated (rodlike) dust grains. We have graphically shown the parametric regimes (the region above the $\Psi = 1$ curves) for which the Mach cones can be formed in laboratory dusty magnetoplasmas.\(^4,5,17,18\) We have found that for waves of fixed wavelength, as we increase the magnitude of $\Omega_r$, we need a dust particle of higher speed in order for the creation of the Mach cones. We also observed that the Mach cone opening angle $\theta$ decreases with the wavelength, but increases with the magnitude of the external magnetic field strength as well as with the dust dipole oscillon frequency $\Omega_r$. We finally propose to conduct laboratory experiments for verifying our theoretical prediction.

ACKNOWLEDGMENTS

This research was partially supported by the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich 591 and by the European Commission through Contract No. HPRN-CT-2000-00140.